#### SIMULATION OPTIMIZATION OF GRAZING MANAGEMENT STRATEGIES

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#### ABSTRACT

In farming systems research, simulation is a common investigation tool that enables to study the dynamic behavior of production systems in response to climatic factors and more or less sophisticated management strategies. SEPATOU is one such a simulator that reproduces the functioning of a rotational grazing dairy system. This paper considers simulation optimization as a means to derive the best values of some parameters involved in a management strategy. We evaluate a stochastic optimization algorithm, the Kiefer-Wolfowitz method, based on a stochastic approximation of the objective function gradient. It appears that this approach is reliable. However the algorithm requires a delicate parametrization that is specific to each application.

## INTRODUCTION

Rotational grazing management is a difficult task that dairy farmers are confronted with. In order to gain a better understanding of this problem of decision under uncertainty (mainly due to unpredictability of climate) and to help agronomist researchers to propose management strategies we have developed a software simulator called SEPATOU (Cros et al., 1999). The software simulates the daily dynamics of a dairy production system composed of a set of grazing fields and a cow herd. The system is governed by technical management operations decided on the base of a strategy. The system is also influenced by climatic factors. The simulator entries are a farm description, a climatic scenario covering the entire period of simulation and a management strategy consisting of general organization rules, operational rules, state indicators and information gathering procedures. The simulator outputs are numerical and text data visualized with tables, graphs, or domain specific representations as grazing calendars or the daily diet composition per cow. More recently, we have addressed the problem of automatically optimizing a parameterized strategy relatively to a specified numerical criterion using SEPATOU. The simulator plays the role of a stochastic function by running it under various climatic conditions picked randomly via a climate generator.

In a simulation optimization problem, the objective function can only be evaluated by computer simulations less or more expensive to realize. This function, often stochastic in nature, is only an implicit function of decision parameters of the system. As there does not exist an analytical expression of the function, differentiation or exact calculation of local gradients are impossible. Even estimation of approximate local derivatives could be a problem. However a great advantage of this approach is that the complexity of the modeled system does not significantly affect the performance of the optimization process.

In this paper, we describe the Kiefer and Wolfowitz (KW) stochastic optimization method that is a classical and simpler algorithm for solving stochastic optimization problems. Some case studies concerning rotational grazing management are described. Preliminary results are discussed.

#### STOCHASTIC OPTIMIZATION METHOD

A general formulation of the considered problem is to optimize a strategy parameterized with p numerical parameters (possibly continuous or discrete) relatively to a numerical criteria. Let  $\theta$  be the parameters vector and  $\Theta$  be the value domain of  $\theta$ . The criterion is a stochastic function  $J: \theta \to J(\theta), \Theta \subset \mathbb{R}^p \to \mathbb{R}$ . For each simulation, J may have a different value. The problem we consider is to find the value of  $\theta$  named  $\theta^*$  that maximizes (in this study) the expected value of the criterion :

$$\theta^* = \underset{\theta \in \Theta}{\arg\max} E(J(\theta))$$

Different methodologies (Azadivar, 1999) exist to answer this single objective problem. We are interested in evaluating a stochastic approximation method adapted to the stochastic nature of the simulation model.

Stochastic approximation methods are based on the original work by Robbins and Kiefer and Wolfowitz (Kiefer et al., 1952). They are recursive procedures that approach to  $\theta^*$  using noisy observations made on the function *J*. The recursive formula is given as :

$$\theta_{n+1}^{i} = \theta_n^{i} + a_n \cdot \frac{J(\theta_n + c_n \cdot e^i) - J(\theta_n - c_n \cdot e^i)}{2c_n} \quad \forall i = 1, \dots p$$

where  $\theta^i$  is the i<sup>th</sup> component of  $\theta$ ,  $e^i$  is the vector with 1 for the i<sup>th</sup> component and 0 for the others,  $a_n$  and  $c_n$  are two series of positive real numbers that satisfy the following conditions :

$$\sum a_n < \infty$$
,  $\lim_{n \to \infty} c_n = 0$  and  $\lim_{n \to \infty} (\frac{a_n}{c_n})^2 < \infty$ 

It has been proven that  $\theta_n$  converges to  $\theta^*$ .

For our problem, the following common values were chosen for the series:

$$a_n = \frac{a}{n}$$
 and  $c_n = \frac{c}{n^b}$ 

where a > 0, c > 0 and  $b \in [0; 0.5[$ .

Considering our application, parameters could be quite different with respect to the size of their respective domains. We thus divide for each parameter  $\theta^i$  the  $a_n$  and  $c_n$  values by the size of the  $\theta^i$  domain. Moreover we force by a projection  $\theta_n$  to stay in  $\Theta$ .

The initial point of the algorithm is explicitly or randomly defined. The algorithm is stopped when a specified number of iterations is reached or if the mean on several iterations of the maximum component gradient value is less than a specified threshold.

## EXPERIMENTS

## **Case studies**

In order to evaluate the method some experiments were conducted by exploring some hypothetical grazing management problems in the Ségala region (located in Aveyron, in south-west of France). We have considered three prototypical cases, each characterized by a farm configuration and a particular management strategy as follows:

- *Case A*: 22 are/cow, that is 6 fields of 1 ha each and 30 cows, calving the 1<sup>st</sup> October of the previous year, 30 t of maize silage, high nitrogen supply, stop of use of conserved feed in spring;
- *Case B* (more extensive than Case A): 30 are/cow, that is 6 fields of 1.5 ha each and the same herd characteristics than in Case A, 35 t of maize silage, low nitrogen supply;
- *Case C* (earlier turn out than Case A): 30 are/cow, that is 6 fields of 1.5 ha and the same herd characteristics than in Case A, 25 t of maize silage, high nitrogen supply, turnout to grazing as early as possible.

For these strategies (see Cros et al., 1999 for details), nine parameters were considered for simulation optimization experiments:

- the threshold *BG* of dry matter quantity required per cow in order to enable turnout to grass (expressed as the number of days of entirely herbage-based feeding, *BG* in [3; 15]);
- the threshold *DMleft* of dry matter quantity to be left on the field when living it (*DMleft* in [100; 300] g/m2);
- the lengths *d1* and *d2* of the first and second stages of reduction of maize silage complementation (expressed in number of days, *d1* in [5; 45], *d2* in [5; 10]);
- the quantities q1 and q2 of maize given to a cow per day in the above two stages (q1 in [7; 12] kg, q2 in [3; 7] kg);
- the first and second length extensions, d2' and d2", of the second stage of reduction of maize complementation (d2' and d2" in [5; 20] days);

- the third nitrogen supply *N3* (expressed qualitatively, in {low, medium, high}). The parameters have been optimized one by one, by cluster of three of them or all together. Several criteria with different sensibility to the climate were considered: *total quantity of milk per cow, total amount of herb intake*, etc.

A climate generator inspired from (Rackso et al., 1991) was used to generate climatic years (temperature, rain, radiation, PET) on the basis of several real climatic years collected in the location considered.

## Algorithm parameterization

Three parameters have to be specified for the algorithm. The parameter *a* influences the distance between  $\theta_{n+1}$  and  $\theta_n$ . If it is too small the algorithm does not explore the search space sufficiently. If it is too high, the algorithm converges too slowly. The parameter *b* influences the speed of decrease of the interval size used to estimate the stochastic gradient;

the smaller *b*, the slower the interval size reduction. Finally the parameter *c* determines the initial size of the considered interval to estimate the stochastic gradient; the smaller *c*, the smaller the interval size. Some experiments were necessary to parameterize appropriately the algorithm for a specific problem. In case of bad parameters, the convergence could be too fast (convergence in direction of a near local optimum depending on initial values) or too slow (even after 50,000 iterations the algorithm may still be oscillating between min and max values). In our case, a = 5, b = 0.1 and c = 1 were often empirically found to be good values.

The objective function evaluation is in general very dependent on the climate: the standard deviation divided by the average of the objective function value varies vary from 0.02 to 0.20. It seems better therefore to evaluate the criterion by averaging the values obtained for several, say m, climates. At each iteration of the algorithm,  $m \cdot 2p$  simulations are then necessary instead of 2p but this brings a better approximation of the stochastic gradient and the algorithm convergence is faster. Taking m = 5, was generally found to result in a good compromise between the time spent to improve the gradient evaluation and the gain in speed of convergence of the algorithm.

Finally it is necessary to specify a stopping condition. We always preferred to specify a limit on the number of iterations. It was then possible to visualize the evolution of the algorithm, to monitor the convergence and eventually to carry on with the search if necessary.

# **RESULTS AND DISCUSSION**

We observed that, as expected, despite noisy observations due to the stochastic nature of the simulation model the algorithm could indeed approach fairly closely the optimum values searched for. In each of the 3 cases A, B, C, a large number of iterations were needed: up to 100.000 iterations were done to optimize the nine parameters. The values reached were good values, except when the algorithm was really badly parameterized. Sometimes, it was hard to know if the algorithm has converged or if more iterations should be done.

To have an estimation of the quality of the optimum found with the stochastic optimization method, a discretization-enumerative algorithm was used. This algorithm explores, after discretization, all the possible values of the parameters and assesses their worth at each point by averaging the criterion computed for different climatic scenarios. For each point, up to 500 simulation (depending of the size of the search space) were done. This was realizable because of the little number of parameters to optimize (up to nine). We noticed that the enumerative algorithm could be quite attractive because it allowed to apprehend the general shape of the objective function (e.g. large numbers of local optimal values) and the noise on the observations (how many simulations are necessary to see a curve to appear, ...). In our case, the enumerative algorithm could be a good tool for optimizing strategies involving a small number of parameters.

Let us present briefly the kind of results we obtained with the KW algorithm. Consider the case C with the criterion *total quantity of milk per cow*. The figure 1 shows the evolution of the criterion value as a function of the number of iterations when optimizing *BG*, *DMleft* and *d2*. We see that the convergence is quite fast and that 4,000 iterations seem to be sufficient. Note the fluctuating behavior of the objective function values.

On Figure 2, the evolution of the *DMleft* parameter is plotted for the same run than in Figure 1. The convergence of the parameter is slower than in Figure 1. Note that after 4,000 iterations the further convergence of the parameter does not bring any improvement of the criterion.

Considering the nine parameters to optimize, the optimal values found with 150,000 iterations (2,000,000 simulations running for 6 days on a dedicated PC) are described in Table 1. In this table, the optimal values found with the enumerative algorithm (1,234,548 simulations) are also given. The estimated value (with 1,000 climates) of the objective function is 3,743 kg (standard deviation of 46 kg) of produced milk per cow for the KW algorithm and 3,745 kg (standard deviation of 65 kg) for the enumerative one. We note the closeness of the two solutions with respect to the criterion. They were found realistic from an agronomic point of view.

## CONCLUSION

In this study, the optimization of a parametrize rotational grazing strategy was investigated using a stochastic approximation method based on the Kiefer-Wolfowitz method. This method is based on a stochastic approximation of the objective function gradient using a simulator of the application of strategy under various climates. It appears that this approach is reliable. However the algorithm requires a delicate parametrization that is specific to each application.

For future works, rather than exploring other types of algorithms, we feel that it would be worthy to work on the expression of the criteria (possibly multiple objectives) and on the evaluation of the optimum. That is, we addressed here the problem of optimizing the expected value of an objective function. This however, does not respond to all interesting and practical questions. The optimum might be differently evaluated for example when the risk exceeding a certain threshold or when the dispersion of the response are minimized.

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Algorithm	BG	DMleft	dl	d2	<i>q1</i>	$q^2$	d2'	d2''	N3
	(days of	$(g/m^2)$	(days)	(days)	(kg per	(kg per	(days)	(days)	
	grass per				cow)	cow)			
	cow)								
KW	12	102	36	5	9.5	6.3	9	7	high
enumerative	6	150	38	5	7	5	5	5	high

TABLE 1. Optimal values found with the KW algorithm and the enumerative one in the case C with all the nine parameters to optimize for the criterion : *total quantity of milk per cow* 

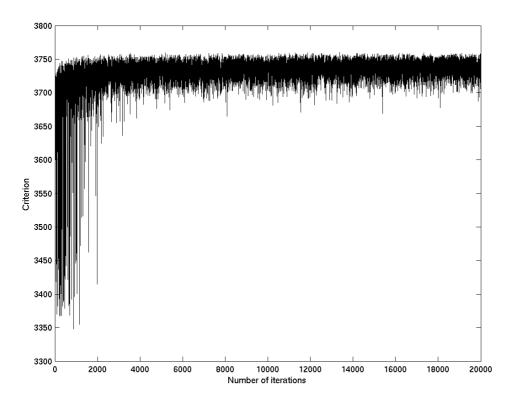


FIGURE 1. Evolution of the criterion : *total quantity of milk per cow* in function of the number of iterations in the case C with the parameters *BG*, *DMleft* and *d2* to optimize.

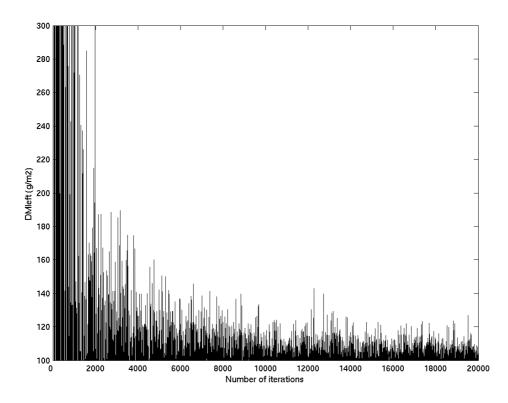


FIGURE 2. Evolution of the parameter DMleft (in g/m<sup>2</sup>) in function of the number of iterations in the case C with all the parameters BG, DMleft and d2 to optimize for the criterion: total quantity of milk per cow