

# **SIMULATING MANAGEMENT POLICIES ON STOCK SUPPLIED BY MULTIPLE PRODUCTION UNITS: APPLICATION TO A PIG SLURRY TREATMENT PLANT**

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## **ABSTRACT**

Because of intensive animal farming, the problem of animal wastes disposal has become very acute in the Reunion Island, namely for liquid wastes such as pig slurry. In this paper we address the problem of determining efficient supply policies of a slurry treatment plant collectively managed by individual farmers. For this, we built a simulation model with a continuous part (i.e., slurry stocks modeled as ordinary differential equations) and a discrete part (i.e., transport allocation as a Linear Program). Simulations are carried out considering the specific constraints of farmers in a critical zone in the Reunion Island. The results emphasize the positive role of a closed-loop control, which means organizing slurry transportation according to the current states of individual stocks.

## **INTRODUCTION**

With the development of intensive animal farming in the highlands of the Reunion Island in the past ten years, the problem of animal wastes disposal has become more acute, namely for liquid wastes such as pig slurry. In some places like Grand-Ilet, the available cultivated area is far too small to allow pig slurry spreading on crops in conditions complying with the regulation [Renault and Paillat, 1999]. This results in an excessive crop fertilization and related risks for the environment. Therefore, the treatment of pig slurry appears as a necessity in this area. Since all of the 56 pig farms of Grand-Ilet are very small (compared to European standards) and cannot afford individually the treatment costs, the choice is being oriented towards a collective solution. In this paper we address the problem of determining efficient supply policies of such a collective slurry treatment plant (STP) by the individual farms. Questions to be answered are typical of stock management problems: Who must deliver pig slurry to the STP? When? How much must be delivered?

We present first, the system entities and describe the different types of supply policies to be envisaged. The criteria to compare alternative policies are the minimization of slurry stock overflow (losses to the environment) of both the farms and the STP. Then, we describe a conceptual view of the model as a block-diagram and its mathematical formalization made of:

- a continuous part, as ordinary differential equations accounting for the dynamics of the stocks;

- a discrete part, as a Linear Program [Wolsey, 1998] representing the transport organization to deliver slurry to the STP.

Finally, two simulation scenarios, based on the specific constraints of Grand-Ilet farms, are presented.

## APPLICATION BACKGROUND

### System description

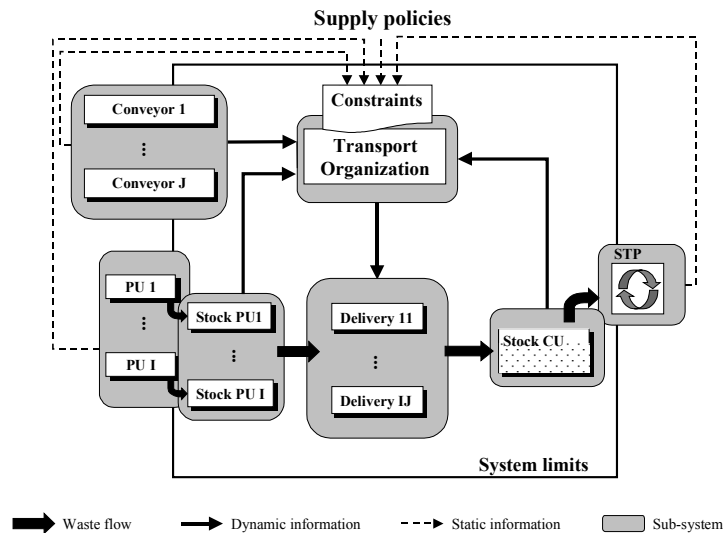


FIGURE 1. Overall system made of many individual pig farms and a collective slurry treatment plant.

Figure 1 provides the reader with a functional view of the system to be modeled. The output stocks of a set of production units ( $PU_i$ ) accounting for the individual farms are considered with the input stock of a unique consumption unit (CU) accounting for the STP. The STP itself, where the transformation process of input matter occurs, is not considered. The *Transport Organization* module acts as the system controller. It is aimed at allocating *Conveyors* (acting as system actuators) to carry slurry from the PUs to the CU, according to various constraints, namely those defining alternative supply policies. In order to take into account changes in stock evolutions (that may be an important decision criterion) this model must be dynamic.

### Supply policies

Whatever the application domain, stock management problems involve questions such as: Who should supply the stock? When should it be done? For how much? [Giard, 1988]. The first two questions may be aggregated as follows: "When should each  $PU_i$  perform a delivery?". There are two classical alternatives to answer the *When?* question:

- T policy: the supply date is determined on a fixed period basis (e.g., the number of weeks between two deliveries);
- S policy: deliveries are triggered when the stock state passes an alarm threshold.
- Two policies can also be considered to answer the How much? question:
- Q policy: a fixed, predetermined, quantity is delivered at each time;
- R policy: a variable quantity is delivered in order to move the stock state back to a predetermined level (e.g., refilling a tank).

Coupling both sets of answers, we get four complete policies able to answer both questions: (T,Q), (S,Q), (T,R), (S,R). Note that, since R is generally determined as S+Q, a (S,R) policy gives a result similar to a (S,Q) policy. Thus, we consider in the following the (T,Q), (S,Q) and (T,R) policies only. Note also that these policies may be applied to the PUs as well as to the CU. For the latter, in fact, we will assume, here, that deliveries should be achieved on a daily basis, 220 days per year. Therefore, policies Q and R only are to be considered for the CU.

## MODEL DESCRIPTION

### Block diagram

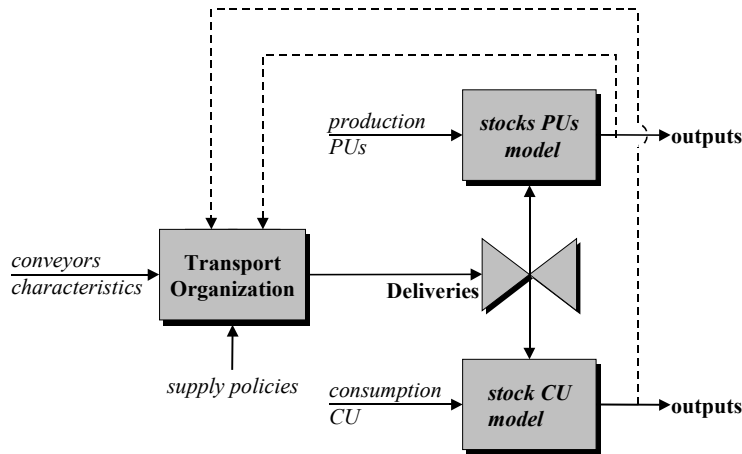


FIGURE 2. Block diagram representation of the CU supply system.

The block diagram representing the CU supply system is shown on Figure 2. We have two distinct processes to be jointly controlled: process #1 made of the PUs stocks, and process #2 corresponding to the CU stock. Transport actions performed by the conveyors directly act upon these stocks and are considered as the system actuators.

The set of the  $(i,j)$  transport configurations can be viewed as a set of  $(i,j)$  valves with two discrete on/off states, to connect process #1 to process #2 and make pig slurry flow from the PUs to the CU. As outlined before, the Transport organization module is the system's controller. The system's outputs are the states of both processes.

A (T,Q) policy can be viewed as an open-loop control, as the states of the processes are not taken into account to determine transport actions. In contrast, (S,Q) and (T,R) policies are viewed as closed-loops, since both S and R need the stock states to be known.

Here we get a *Hybrid Dynamical System* [Antsaklis et al., 1998] that is decomposed into two parts: a continuous part, made of processes #1 and #2 (since the variables representing stock evolutions take value on the real set), and a discrete part made of the transport organization controller (transport actions are Boolean variables with on/off values).

### Stock evolutions: the continuous part

The rate of change of any stock is modeled as an ordinary differential equation according to the mass balance equation:

$$\frac{dV}{dt} = \sum_{m=1}^M Q_{in_m} - \left( \sum_{n=1}^N Q_{out_n} + Q_{over} \right) \quad (1)$$

$$V(0) = v_0 \quad (2)$$

Where  $V(t)$  is the volume of the stock,  $Qin_m$  ( $m = 1, \dots, M$ ) and  $Qout_n$  ( $n = 1, \dots, N$ ) are respectively the stock inflows and outflows, and  $Qover$  stands for the overflow according to the maximum stock capacity ( $v_{max}$ ). It is defined by:

$$Qover = \begin{cases} \sum_{m=1}^M Qin_m - \sum_{n=1}^N Qout_n & \text{if } V \geq v_{max} \quad \text{and} \quad \sum_{m=1}^M Qin_m > \sum_{n=1}^N Qout_n \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

### **Transport organization : the discrete part**

Each production unit  $PU_i$  ( $i = 1, \dots, I$ ) may use a conveyor  $C_j$  ( $j = 1, \dots, J$ ) which can at most perform  $k = 1, \dots, K$  daily roundtrips between a PU and the CU. Optimizing the system would be to achieve as many trips as possible while satisfying the organizational coercions (e.g. supply policies) and resource constraints (e.g. daily available working time of conveyors, volumes of PU and CU stocks, carrying capacity of conveyors and limits put on assigning conveyors to PUs).

The transport organization module has been computed as a Linear Program (involving integer variables) in which any  $x_{ijk}$  stands for a decision variable for performing delivery trips. This program is executed every day for which transportation is possible. The combinatorial feature of the bivalent nature of variables  $x_{ijk}$  is classically handled by tree-search procedures such as Branch and Bounds methods. The objective function to be maximized is the number of delivery trips from the PUs to the CU (i.e., the slurry flow between the farms and the STP) with a cost  $Pw \in [0,1]$  accounting for a delivery time indicator in the case of policies T or S applied to the PUs:

$$Max \sum_{ijk} Pw_i \cdot x_{ijk} \quad (4)$$

The constraints are put in the form of inequalities. For example, the constraint holding for the volume of PUs is as follows:

$$\sum_{jk} vc_j \cdot x_{ijk} \leq Ph_i \quad (5)$$

with:  $vc_j$  the carrying capacity of conveyor  $j$  and  $Phi$  the upper bound for the quantities to be delivered defined by the policies Q or R applied to  $PU_i$ .

The constraint stating that any conveyor cannot be at different places at the same moment is:

$$\sum_i x_{ijk} \leq 1 \quad \forall j, k \quad (6)$$

### **SIMULATION**

Our model was implemented with the Matlab/simulink software and parameterized with data relevant to the case of Grand-Ilet farms (e.g., transport unitary volume  $\leq 10$  m<sup>3</sup>). Performing simulations allowed us to check various policies, different sizes for PU stocks as well as varying the production capacity of the PUs on a yearly basis. We also considered farmers preferences (e.g., some are likely to perform slurry transportation by themselves whereas others are dependent upon a collective conveyor).

We show, here, the simulation outputs of two scenarios involving both a collective conveyor with 10 m<sup>3</sup> carrying capacity and 40 PUs with 350 m<sup>3</sup> stock capacity. Two characteristic policies are tested: (T,R) and (S,R), respectively. Initial values for stocks are half the full stock capacity for the CU and a level randomly assigned to each PU (though it remains the same for every simulation).

## Scenario 1

The policy applied to all the PUs is (T,R), i.e. with planned delivery dates (delivery time T is fixed) and aiming at emptying the stocks (R is the current volume of any stock). The policy for the CU is of type Q, i.e. a fixed quantity should be delivered each day (the quantity of slurry processed each day). Figures 3 and 4 illustrate the stock evolutions of the PUs and the CU, respectively. The volumes are normalized to 1, i.e. given as the ratio current volume/stock capacity. Although the mean value tends to decrease regularly, one can notice that some PUs are regularly overflowing (cf. Fig. 3 where the max value is almost always at 1): losses to the environment represent a total quantity of 1800 m<sup>3</sup> (i.e. Qover integration over time). Similarly, Figure 3 exhibits that the same happens sometimes to the CU.

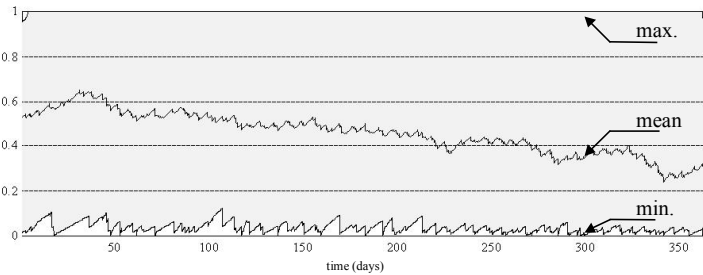


FIGURE 3. Min, mean, and max values of normalized stock evolutions for the PUs in scenario 1.

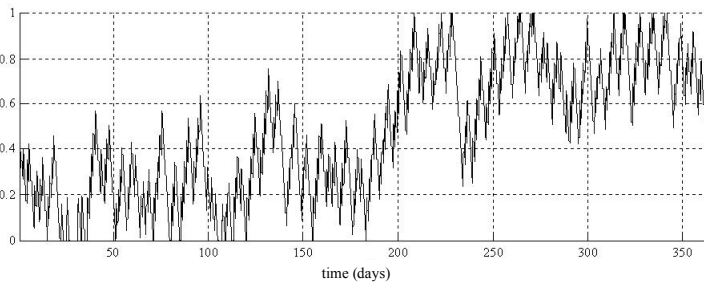


FIGURE 4. Normalized stock evolution of the CU in scenario 1.

## Scenario 2

The policies simulated here are (S,R) for the PUs, with S defined as the maximal normalized states of stocks and R their current volume. For the CU a R policy is taken (with aim to refill the stock up to its limit capacity). Figures 5 and 6 illustrate the stock evolutions for the PUs and the CU, respectively. We get here no slurry overflow to the environment and thus, obtain a much better regulation than in the first scenario, due to a better use of available resources (conveyors).

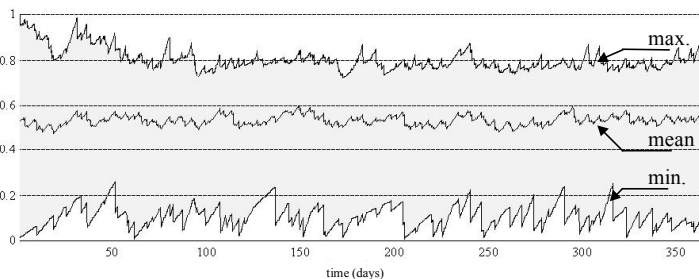


FIGURE 5. Min, mean, and max normalized values of stock evolutions for the PUs in scenario 2.

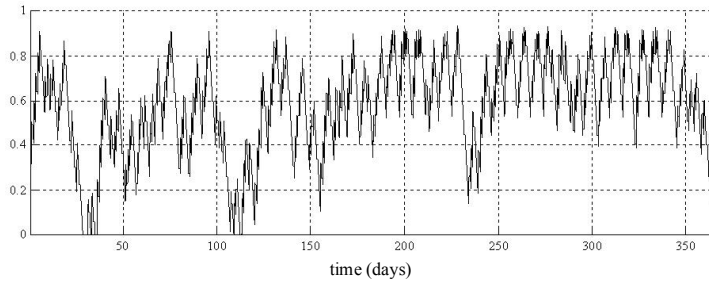


FIGURE 6. Normalized stock evolution of the CU in scenario 2.

## SUMMARY AND CONCLUSION

The model described in this paper belongs to the category of Hybrid Dynamical Systems as it embodies both:

- a continuous part, representing the stock evolution of production and consumption units, and encoded as ordinary differential equations;
- a discrete part, representing the transport organization, implemented as a Linear Program, and acting as the delivery system controller.

Beyond the two simulations presented here, many others, accounting for different policies and various parameter values, were also studied on the specific case of Grand-Ilet farms. They have demonstrated the great benefit that can be gained for controlling the system if a sharp reactivity in deliveries could be installed. This corresponds to a closed-loop control where actuators (conveyors) are triggered according to the current states of stocks (like in scenario 2). We are now trying to generalize this approach by devising a generic production/consumption model that could be used, by means of model composition techniques, to represent different cases of stock problems relevant in the framework of animal wastes management (e.g., decentralized delivery from many PUs to many CUs). For this we are further investigating the methods taken from the domains of Hybrid Dynamical Systems and Scheduling under time and resources constraints [Lopez and Roubellat, 2001] [Pinedo and Chao, 1999].

## ACKNOWLEDGEMENT

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## REFERENCES

- Antsaklis P., Koutsoukos X., Zaytoon J. (1998) On hybrid control of complex systems: a survey, *APII – JESA*, vol. 32, n°9/10, 1023-1045.
- Giard, V. (1988) *Gestion de production*, Economica, coll. Gestion, Paris, 2<sup>nd</sup> edition.
- Lopez P., Roubellat F. (Eds.) (2001) *Ordonnancement de la production*, Hermès Sciences Publications, Paris, 2001.
- Pinedo M., Chao X., (1999), *Operations scheduling with application in manufacturing and services*, Irwin/McGraw-Hill, Boston.
- Renault D., Paillat J.-M. (1999) *Analyse de la production et de l'utilisation des effluents porcins à Grand-Ilet, localité de l'île de la Réunion (cirque de Salazie)*, Cirad Tera/Ere, rapport 16/99, La Réunion.
- Wolsey, L.A. (1998) *Integer programming*, Wiley, New York.