

# QUALITATIVE DYNAMIC PROGRAMMING FOR OPTIMAL TIMBER CUTTING AND CONSERVATION OF LEADBEATER'S POSSUM

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## ABSTRACT

Leadbeater's possum is an endangered species particular to mountain ash country in the Central Highlands of Victoria. The species is threatened by the outbreak of catastrophic wildfire and the loss of nesting sites resulting from logging trees younger than 150 years of age. A stochastic dynamic programming approach to determine how much logging should be foregone to enhance the probability of survival of the possum rests on estimates of the probability of wildfire and the value of preservation of the possum. Given the difficulty of obtaining precise estimates, solution methods using qualitative, fuzzy estimates are explored.

## INTRODUCTION

Leadbeater's possum is an arboreal marsupial thinly scattered in the montane ash forest of the Central Highlands of Victoria, Australia. The species relies on access to trees which are typically at least 200 years of age with hollows sufficiently large for nesting. Current logging rotations of about 80 years threaten the survival of the possum. It is officially listed as an endangered species and is listed under Victoria's Flora and Fauna Guarantee Act. Leadbeater's possum has a special status with the Victorian public in that it is one of Victoria's two faunal emblems. It is clear that logging practices which would maximise the probability of survival of Leadbeater's possum are different from those which would maximise the net present value of timber production alone. Questions arise as to whether logging should be excluded from some parts of the forests which are particularly favourable habitat for the possum, or whether it is both feasible and economic to modify logging practices to increase the probability of survival of the possum whilst still obtaining a return from timber. The problems to be considered are, for a homogeneous area of forest, what would be the optimal age at which to cut trees, and whether any cut should be total or partial. Stochastic events are possum survival and the outbreak of a wildfire. Logging decisions have to be taken now which may have irreversible consequences for the possum, taking account of logging decisions and the risks of species extinction and of wildfire in all future periods.

## A STOCHASTIC MARKOV DECISION PROCESSES (MDP) APPROACH

### **Specification [Kennedy, 1999]**

The objective is to find the sequence of harvesting decisions made at 50-year intervals which leads to the maximum expected present value of returns from timber production and habitat

preservation in all future 50-year periods. Land is kept perpetually forested. Either all trees are the same age, or 10 per cent of trees have been saved from previous clearfelling in order to develop a stock of hollow-bearing trees for nesting sites and are therefore older. The latter trees are referred to as being in the ‘old’ 10 per cent class, and aged  $a_{10}$ . The remaining ‘new’ trees in the 90 per cent class are aged  $a_{90}$ . One of three harvesting decisions is made at each decision stage: clearfell all trees ( $d=1$ ); clearfell only the ‘new’ 90 per cent category of tree ( $d=2$ ); or leave all trees standing ( $d=3$ ). The harvesting decision is implemented 50 years after the decision has been taken, just before the next decision stage. Two stochastic events may occur over the 50 years between one decision stage and the next. One is an intense wildfire which destroys much of the forest ( $i=1$ ). All of the trees in the 10 per cent class are destroyed, and enough of the trees in the 90 per cent class survive after salvage operations to become a new 10 per cent class if older than 50 years. Any fire occurs immediately before a harvesting decision would have been implemented. The way in which the age of trees in each category is determined for the next decision stage after 50 years is shown in Table 1.

TABLE 1: Tree age by tree class after 50 years.

Tree class	i=0 (no fire)			i=1 (fire)		
	Cut decision			Cut decision		
	d=1	d=2	d=3	d=1	d=2	d=3
	100%	90%	0%	100%	90%	0%
‘New’ (90%)	0	0	$a_{90}+50$	0	0	0
‘Old’ (10%) $a_{90}=50$	0	$a_{10}+50$	$a_{10}+50$	0	0	0
‘Old’ (10%) $a_{90}>50$	0	$a_{10}+50$	$a_{10}+50$	$a_{90}+50$	$a_{90}+50$	$a_{90}+50$

The other stochastic event is survival of the possum to the next decision stage. The probability of survival, shown in Table 2, is greater the older the trees in each class, if there is no harvesting, and if there is no wildfire. Obviously survival to the next decision stage is only possible if the possum has survived to the current decision stage.

TABLE 2 Probability that Leadbeater’s possum survives to the next decision stage.

Tree Age (Years)		i=0 (no fire)			i=1 (fire)
		d=1	d=2	d=3	All d
90%	10%	100%	90%	0%	
$a_{90} < 200$	$a_{10} < 200$	0.000	0.000	0.000	0.000
$a_{90} < 200$	$a_{10} \geq 200$	0.000	0.125	0.250	0.125
$a_{90} \geq 200$	$a_{10} \geq 200$	0.000	0.410	0.820	0.410

The return from harvesting is the value of the age-dependent yields of three grades of timber, less regeneration costs for the next cohort of trees. If wildfire occurs, a proportion of all trees is salvaged. The probability of wildfire in any year is estimated to be 0.01, which translates to a probability of at least one fire occurring in 50 years of 0.40. In order to keep the number of states of the system to a minimum, any tree age greater than 300 is classified as 300. Any changes in merchantable volume or nesting site suitability after 300 years of age are insignificant. The expected existence value of Leadbeater’s possum to the Victorian community is calculated as the product of the probability of survival over most of the ensuing 50 years, before cutting or wildfire can occur, and the mean amount Victorians are estimated to be willing to pay for preservation of the species. This assumes a linear relationship between expected existence value and probability of survival.

### **The Markov Decision Processes (MDP) framework**

The standard MDP model [Puterman, 1994] is defined by: set  $T \subseteq \mathbf{N}$  of decision stages for which when  $T = \{0, \dots, N\}$  is finite,  $N$  is the horizon of the problem and for each stage  $t$ , a

finite state space,  $S_t$ . Sets  $A_{s,t}$  (finite) of available actions in state  $s$  at stage  $t$  are defined (these sets are denoted  $A_s$  when they are independent of  $t$ ) as well as rewards  $r_t(s,a,s')$  (that may be negative) that are obtained after  $a$  has been applied in state  $s$  and resulted in  $s'$  and probability distributions  $p_t(\cdot|s,a)$  describing the uncertainty about the possible successor states (in  $S_{t+1}$ ) of  $s \in S_t$  when  $a \in A_{s,t}$  is applied.

A *decision rule*  $d_t$  is an application from  $S_t$  to  $\cup_{s \in S_t} A_{s,t}$  assigning an action to each possible state of the world in stage  $t$ . A *policy*  $\delta$  is, in the finite horizon case, a  $N$ -tuple of decision rules  $\delta = (d_1, \dots, d_N)$ . Let  $\Delta = D_1 \times \dots \times D_N$  be the set of applicable policies.  $\Delta$  is the cross-product of the sets of applicable decisions for each stage. In the stationary infinite horizon case, the parameter  $t$  has no influence on the decision problem. Thus, a policy  $\delta$  is nothing but the repetition of an identical decision rule  $d$ . A policy  $\delta$ , applied in an initial state  $s_0$  (together with the probability distributions) defines a *Markov chain* that describes the sequence of states occupied by the system. The *value of a policy* in a given state is the expected sum of the rewards gained along the possible trajectories. In the finite horizon case :

$$v(s_0, \delta) = E\left[\sum_{t=0..N} \gamma^t \cdot r_t(s_t, d_t(s_t))\right] \quad (1)$$

When the horizon is infinite, the discounted value of a policy is defined by :

$$v(s_0, \delta) = E\left[\sum_{t=0..∞} \gamma^t \cdot r_t(s, d(s))\right] \quad (2)$$

where  $0 < \gamma \leq 1$  is the discount factor.

Solving a MDP amounts to finding a policy  $\delta^*$  maximizing  $v(s_0, \cdot)$ . The *dynamic programming* methods are based on the decomposition of the sequential decision problem into one-stage decision problems, by making use of Bellman's equations [Bellman, 1957]. In the finite horizon case, optimal policies can be computed by the *backwards induction* algorithm which solves the following equations in decreasing order of  $t$ .

$$v_t(s) = \left\{ \sum_{s' \in S_{t+1}} p(s'|s,a) \cdot (r(s,a,s') + \gamma \cdot v_{t+1}(s')) \right\} \quad t = N, \dots, 1 \quad (3)$$

In the discounted infinite horizon case, optimal policies (which are stationary) can be obtained as fixed points of equation (4) below. Methods such as the *value iteration* algorithm or others (see [Puterman, 1994]), can be used to compute optimal policies.  $Q(s,a)$  represents the expected value of performing action  $a$  in state  $s$ . This is to be distinguished from  $v(s)$ , which is the expected value of performing the optimal action in state  $s$ .  $Q(s,a)$  is defined by

$$Q(s,a) = \left\{ \sum_{s' \in S_{t+1}} p(s'|s,a) \cdot (r(s,a,s') + \gamma \cdot v(s')) \right\} \quad (4)$$

and  $\forall s \in S, v(s) = \max_{a \in A_s} Q(s,a)$ .

It is easy to get an optimal, stationary, policy  $\delta^*$  from  $Q$ , since  $\delta^*(s) = \operatorname{argmax}_a Q(s,a)$ .

### **Resolution**

In [Kennedy, 1999], the model parameters are estimated from either economic values (price of timber of different age categories, regeneration costs, discount rate), or questionnaires (willingness of Victoria citizens to pay for the survival of Leadbeater's possum).  $r(s,a,s') = r_c(s,a,s') + r_p(s,a,s')$  is the sum of the return from timber cut and a monetary value attached to possum's survival (each year).

Of course  $r_p(s,a,s')$  can only be estimated approximately. A contingent valuation survey of a large sample of the population resident in Victoria was conducted by questionnaire, (see Jakobsson and Dragun, 1996). Under alternative assumptions, the individual willingness to pay ranged from A\$6.40 to A\$40.04. Of course, the obtained optimal policies depend heavily

on this parameter, as well as on the discount factor  $\gamma$ . Results are listed in table 5. See [Kennedy, 1999] for complete details.

## A QUALITATIVE APPROACH OF THE PROBLEM

### **Possibilistic Markov Decision Processe(PI-MDP) framework**

[Dubois & Prade, 1995] proposed an ordinal counterpart, based on possibility theory, of the expected utility theory for one-stage decision making. In this framework,  $S$  and  $X$  are respectively the (finite) sets of possible states of the world before and after an action is taken ( $S_t$  and  $S_{t+1}$  in the multistage case).  $L$  is a finite totally ordered (qualitative) scale, with lowest and greatest elements denoted  $0_L$  and  $1_L$  respectively.  $L$  will be used for assessing both uncertainty and preference degrees. The uncertainty of the agent about the effect of an action  $a$  taken in state  $s$  is represented by a *possibility distribution*  $\pi(\cdot|s,a) : X \rightarrow L$ .  $\pi(x|s,a)$  measures to what extent  $x$  is a plausible consequence of  $a$  in  $s$ .  $\pi(x|s,a) = 1_L$  means that  $x$  is completely plausible, whereas  $\pi(x|s,a) = 0_L$  means that it is completely impossible. Stage returns are expressed in terms of levels of satisfaction by a qualitative utility function  $\mu : S \times A \times X \rightarrow L$ .  $\mu(s,a,x) = 1_L$  means that applying  $a$  in  $s$  and obtaining  $x$  is completely satisfactory, whereas if  $\mu(s,a,x) = 0_L$ , it is totally unsatisfactory. It should be noted that  $\pi$  is normalized (there shall be at least one completely possible state of the world), but  $\mu$  may not be (it can be that no consequence is fully satisfactory).

[Dubois & Prade, 1995] proposed the two following qualitative decision criteria:

$$Q^{opt}(s_0, a) = \max_{x \in X} \min \{ \pi(x|s_0, a), \mu(s_0, a, x) \} \quad (5)$$

$$Q_{pes}(s_0, a) = \min_{x \in X} \max \{ n(\pi(x|s_0, a)), \mu(s_0, a, x) \} \quad (6)$$

where  $n$  is the order reversing map of  $L$ .

$Q^{opt}$  can be seen as an extension of the *maximax* criterion which assigns to an action the utility of its best possible consequence. On the other hand,  $Q_{pes}$  is an extension of the *maximin* criterion which corresponds to the utility of the worst possible consequence (both  $Q^{opt}$  and  $Q_{pes}$  shall be maximized).  $Q_{pes}$  measures to what extent every plausible consequence is satisfactory, while  $Q^{opt}$  measures to what extent there exists a satisfactory plausible consequence.  $Q^{opt}$  corresponds to an adventurous (optimistic) attitude towards uncertainty, whereas  $Q_{pes}$  is conservative (cautious). In [Sabbadin et al., 1998], [Sabbadin, 2001], the possibilistic qualitative decision theory has been extended to multistage decision making. We have a similar property as in the stochastic case, that is that the optimal possibilistic strategy can be obtained from the solution of the following sets of equations (for all  $s$ ) :

$$Q_t^{*opt}(s, a) = \max_{s' \in S} \min \{ \pi_t(s'|s, a), u_{t+1}^{opt}(s'), \mu_t(s, a, s') \}, \quad (7)$$

$$Q_t^*_{pes}(s, a) = \min_{s' \in S} \min \{ \max \{ n(\pi_t(s'|s, a)), u_{t+1}_{pes}(s') \}, \mu_t(s, a, s') \}, \quad (8)$$

where  $u_{t+1}_{pes}(s) = \max_a Q_{t+1}^*_{pes}(s, a)$  and  $u_{t+1}^{opt}(s) = \max_a Q_{t+1}^{*opt}(s, a)$ .

Equations (7) and (8) extend respectively equations (5) and (6) to the sequential case and allow account to be taken of intermediate degrees of satisfaction, aggregated by a *minimum* along the possible trajectories. Indeed, (7) and (8) can be seen as implementations of a Dynamic Programming approach to the computation of the possibilistic utilities of (multi-stage) policies in the spirit of (5) and (6) :

$$Q^{opt}(s_0, \delta) = \max_{\tau} \min \{ \pi(\tau|s_0, \delta), \mu(s_0, \delta, \tau) \} \quad (9)$$

$$Q_{pes}(s_0, \delta) = \min_{\tau} \max \{ n(\pi(\tau|s_0, \delta)), \mu(s_0, \delta, \tau) \} \quad (10)$$

Where  $\tau$  and  $\delta$  are respectively trajectories and decision rules,  $\pi$  and  $\mu$  being evaluated by the overall minimum of the instantaneous qualitative transition possibilities and returns. Two

possibilistic versions of the value iteration algorithm have been defined in [Sabbadin, 2001]. These algorithms converge to the actual values (optimistic or pessimistic) of  $Q^*$  and  $u^*$  in a finite number of steps.

### **Optimal timber cutting in the possibilistic framework**

**Utility function.** In the possibilistic framework, utility levels are considered as satisfaction degrees (of performing a given action in a given state) rather than additive rewards. These satisfaction degrees will be assumed to be independent of the obtained consequence (thus being of the form  $\mu(s,a)$  rather than  $\mu(s,a,s')$ ). Satisfaction degrees depend both on the survival status of the possum and on the timber production level. States are indeed of the form (Survival,  $a_{90}$ ,  $a_{10}$ ), but in the qualitative approach we consider that  $a_{10}$ , the age of the oldest trees, does not influence the timber revenue enough to be considered in the evaluation. If we assume that uncertainty levels and satisfaction levels are expressed on an ordinal scale  $L=\{0,1,2,3,4,5\}$ , then  $\mu(s,d)$  is represented in table 3, where  $s=(\text{Survival},a_{90})$ .

TABLE 3. Satisfaction degree of cutting  $d\%$  of trees, knowing  $a_{90}$  and Survival.

		Survival=1						Survival=0								
$d \setminus a_{90}$		0	50	100	150	200	250	300	$d \setminus a_{90}$	0	50	100	150	200	250	300
100%		4	4	5	5	5	4	4	100%	2	2	3	3	3	2	2
90%		3	3	4	4	4	3	3	90%	1	1	2	2	2	1	1
0%		1	1	1	1	1	1	1	0%	0	0	0	0	0	0	0

**Possibilistic transition functions.** Let us give the possibilistic transition functions  $\pi(s_j|s_i,d,i)$ . The tables are built empirically from the “qualitative” knowledge about the effects of actions. For all  $a \in A$   $\text{Succ}(a)=\min\{a+50,300\}$  defines the “normal” successor age, when trees are not cut and fire does not occur. Then,  $\pi(a_i'|a_{90},a_{10},d,i)$  can be computed for both categories of trees, decision and fire event:

$$\pi(a_i'|a_{90},a_{10},d,i)=\max \{ \min \{ \pi(a_i'|a_{90},a_{10},d,i=0), \pi(i=0) \}, \min \{ \pi(a_i'|a_{90},a_{10},d,i=1), \pi(i=1) \} \} \quad (11)$$

where  $\pi(a_{90}'|a_{90},a_{10},d,i=0) = 5$  if  $a_{90}' = \text{Succ}(a_{90})$  and 0 if not.  $\pi(a_{10}'|a_{90},a_{10},d,i=0) = 5$  if  $a_{10}' = \text{Succ}(a_{10})$  and 0 if not.  $\pi(a_{90}'|a_{90},a_{10},d,i=1) = 5$  if  $a_{90}' = 0$  and 0 if not.  $\pi(a_{10}'|a_{90},a_{10},d,i=1) = 5$  if  $a_{10}' = \text{Succ}(a_{90})$  and 0 if not.  $\pi(i=0) = 5$  and  $\pi(i=1) = 1$  (we assume that fire is rather unlikely, although not impossible). The possibility  $\pi(s'|s=1,a_{90},a_{10},d,i)$  that Leadbeater’s possum survives next stage is shown below:

TABLE 4 Possibility that Leadbeater’s possum survives to the next decision stage (only if  $a_{10} \geq 200$ .  $\pi(s'=1)$  is uniformly 0 if  $a_{10} < 200$ )

		i=0 (no fire)			i=1 (fire)
$\pi$	$d$	d=1	d=2	d=3	All d
		100%	90%	0%	
$\pi(s'=1)$		0	5	5	5
$\pi(s'=0)$		5	2	1	2

The overall transition function is defined by :

$$\pi(s',a_{90}',a_{10}'|s,a_{90},a_{10},d,i) = \min \{ \pi(a_{90}'|a_{90},a_{10},d,i), \pi(a_{10}'|a_{90},a_{10},d,i), \pi(s'|s,d,i) \} \quad (12)$$

### **Results and interpretation.**

The pessimistic and optimistic possibilistic optimal policies can be computed through the previous section’s mechanism and are represented in table 5. Note that the pessimistic approach amounts to considering the extinction of Leadbeater’s possum as inevitable and thus neglects it to the benefit of the pure maximization of timber revenue, as would the

stochastic approach with a low value for possums. The optimistic approach leads to an optimal approach which is not obtained in any of the configurations considered in the stochastic approach. Intuitively, it amounts to considering that 10% of the forest should be reserved as a sanctuary in which satisfactory conditions are maintained for the possum. Although original, this policy is an hybrid between the stochastic ones obtained for  $W=A\$40.04$  and rates of discount  $r=2\%$  and  $r=4\%$  respectively.

TABLE 5: Optimal cutting strategies :  $W$  = willingness to pay for Possum's survival, per inhabitant.  $r$  = interest rate assumed for the future (discount rate =  $1-r$ ).

State	S	Decision - percentage cut after 50 years						Possibilistic case	
		Stochastic case			Possibilistic case			pessimistic decision	optimistic decision
Age of Trees		$W=\$6.30$			$W=\$40.04$				
		$r=4\%$	$r=2\%$	$r=0\%$	$r=4\%$	$r=2\%$	$r=0\%$		
$\forall a_{90}, a_{10}$	0	100	100	100	100	100	100	100	100
$a_{90}=0; a_{10} \geq 200$	1	100	100	100	100	90	0	100	90
$a_{90}=50; a_{10} \geq 200$	1	100	100	100	100	90	90	100	90
$100 \leq a_{90} \leq 150; a_{10} \geq 200$	1	100	100	100	100	90	0	100	90
$200 \leq a_{90} \leq a_{10}$	1	100	100	0	90	0	0	100	90

## CONCLUSION

In this paper we have proposed a qualitative approach to multistage decision making under uncertainty, based on possibility theory. This kind of approach is especially suitable for problems in which data (preferences, probabilities of transition) are missing or can only be qualitatively estimated. It has been applied to a forest management problem where it has been compared to a stochastic approach with estimated data.

What is most noticeable is that the results obtained in the possibilistic approach, although different from those of the stochastic approach, are still comparable and seem quite reasonable, for a considerably lower effort devoted to parameter estimation. Moreover, the qualitative approach is very general, and may be applied to a wide range of problems considering natural resources, in which data estimation is a difficult problem and reasoning with qualitative/approximate data is useful.

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