SUSTAINABILITY ANALYSIS FOR A FORESTRY MANAGEMENT MODEL

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ABSTRACT

This paper deals with sustainability in forestry management. We study a dynamic control system within the mathematical framework of viability analysis. In particular, we use the concept of viability kernel to reveal sustainable harvesting policies and sustainable resource states. An example illustrates the general statements.

1. INTRODUCTION

Forest perception has always fluctuated according to circumstances and throughout history. Men in « civilized » world rapidly aimed at maintaining constant the forested areas, and once the surface stabilized, at preserving the trees in a structure in age classes so that year after year, the demand in goods (and among them the different wood categories) and services may be satisfied.

This concern about sustainability emerged rather early, as well in day to day management decisions as in forestry literature. The risk involved by such a concept is straightforward, in the forest sector at least: Because of the slow growth of the trees, this would imply to maintain constant some parameters, which leads to a completly different set of forest values or of forest management instructions, and particularly to the search of the Maximum Sustainable Yield (MSY, Hartig, 1796). Samuelson (1976) exposed the consequences of such a solution. And its relation with Faustmann's calculus are now well documented (Terreaux, 1995, Terreaux, 1996 and Rapaport and al., 2000). But the MSY, even if not put into effect, constitutes for many managers the ideal goal to reach.

Moreover, it is clear that traditional forest management methods, along with most of the concepts linked to sustainability (Faucheux and O'Connor, 1998), lead to admit only a single optimal trajectory, which indeed maximizes the determined objective, but on which we do not know anything concerning its flimsiness towards for examples risks or parameters non explicitly inserted in the models.

Here we do prefer have recourse to the viability concept, which consists in the definition of a set of constraints for the state and for the command variables (for example a minimal annual harvest volume), and which aims at each time to be in a state that allows to comply with these constraints in the future. More specifically, we address the following questions:

- Given a sustainable harvesting value level, what are the initial resource conditions for which there exists a sustainable management?

- Given a total forest surface, what is the largest minimal harvesting value for which there exists at least one initial resource level (the initial state of the forest) and an associated sustainable policy?

- What are the possible sustainable policies associated with these sustainable states?

Dealing with this issues, we thus provide an intrinsic characterization of the dynamics with respect to the satisfaction of a constraint of sustainability, without having to consider any criterion to be optimized.

2. THE MODEL

We consider a forest which structure in age is represented at discrete time $t \in \aleph$ by a vector x of $\Re_+^n$: $x(t)=(x_n(t),x_{n-1}(t), \ldots, x_1(t))'$, where $x_j(t)$, j=1…n-1, represents the surface bearing trees whose age, expressed in the unit of time used to define t, is j-1 at time t. The last layer $x_n(t)$ represents the surface older than n-1 at time t.
We assume that the natural evolution (i.e. under no exploitation) of the vector x(t) is described by a linear system of the type: $x(t+1)=Ax(t)$, $x(0)=x^0$ where the matrix A is a matrix of the Leslie type:

$$A = \begin{pmatrix} a_n & a_{n-1} & 0 & \ldots & 0 \\ 0 & 0 & a_{n-2} & \ldots & 0 \\ & & & \ldots & 0 \\ 0 & & 0 & \ldots & a_1 \\ b_n & b_{n-1} & & & b_1 \end{pmatrix}$$

and where a_i, b_i are non-negative parameters.

Assumption A1: We assume that the matrix $A=[A_{i,j}]$ is such that :

$$A_{i,j} \in [0,1], \forall i, j = 1,\ldots n, \quad \text{et} \quad \sum_{k=1}^{n} A_{k,j} = 1, \ \forall j = 1,\ldots,n$$

Then, the free dynamics of the resource (i.e. under no exploitation) ensures that the vector x(t) stays non-negative and the invariance of the total surface:

$$x_i(t) \geq 0, \quad i= 1, \ldots, n \quad \text{and} \quad \sum_{i=1}^{n} x_i(t) = S, \ \forall t \geq 0$$

Now we describe the exploitation of such a forest resource x. We assume the following main hypotheses:

1. The minimum age at which it is possible to cut trees is n. This assumption is made only in order to simplify the representation. Taking into account wood market values, it would be possible to make more precise but more complicate models.

2. Each time a plot is cut at time t, it is "immediately" (i.e. within the same unit of time) replanted, then bearing trees of age 0.

Thus let us introduce the scalar variable decision h(t) that represents the surface harvested at time t. Previous assumptions induce the following controlled evolution:

(E) $x(t+1) = Ax(t) + Bh(t)$,

where B is equal to the column vector (-1 0 … 0 1)'. Since one cannot harvest more than what exists, the decision or control variable h(t) is subject to the constraint:

(R) $0 \leq h(t) \leq CAx(t),\ \forall t \geq 0$ where the raw vector C is equal to (1 0 0 … 0).

Notice that under Assumption A1, for any control law h(t) that fulfills the constraint (R), the vector x(t) remains non-negative and the total surface of the forest stays constant equal to S.

Moreover, to encompass the economic or social feature of the exploitation, we associate the harvesting h(t) with an income and we require this harvesting or equivalently the revenue to exceed some minimal threshold $\underline{h}$ at every time t:

(S) $h(t) \geq \underline{h},\ \forall t \geq 0$

Under this context, we propose the following definitions:

1. Given an initial resource $x(0)=x^0$ and a minimal harvesting threshold $\underline{h}$, a harvesting policy h(0), h(1), ... is **$\underline{h}$-sustainable** from x^0 if the policy h(.) combined with x(.), the solution of dynamics (E) starting from x^0, fulfills the constraints (R) and (S).

2. Given a minimal harvesting threshold $\underline{h}$, an initial resource x^0 is **$\underline{h}$-sustainable** if there exists a $\underline{h}$-sustainable harvesting policy from x^0.

3. A SUSTAINABILITY ANALYSIS

Let us introduce the concept of "viability kernel" (Aubin, 1991):

$$Viab_{\underline{h}} = \left\{ x^0 \in \Re^n \text{ / there exists a sustainable policy from } x^0 \right\}.$$

In this way we can reformulate the questions introduced in section 1:

1. Find the largest value $\underline{h}^*$ for which $Viab_{\underline{h}^*}$ is not empty.
2. Compute the viability kernel $Viab_{\underline{h}}$ for $\underline{h} \geq \underline{h}^*$.
3. Characterize the harvesting policies $\underline{h}$-sustainable from $x^0 \in Viab_{\underline{h}}$.

We define the matrices T, O and the vector 1' respectively by:

$$T = \begin{pmatrix} 0 & & 0 \\ & \dots & \\ 1 & & \end{pmatrix}, \quad O = \begin{pmatrix} C \\ CA \\ \dots \\ CA^{n-1} \end{pmatrix}, \quad 1' = (1,\dots,1)$$

and consider the cases of matrices A and B that fulfill the following assumption:

Assumption A2:

i) The matrix O is full rank,
ii) The vector $1'O^{-1}$ is non-negative,
iii) The vector OB is non-positive.

Define the following felling threshold (which is well defined under Assumption A2):

$$\underline{h}^*(S, A) = \frac{S}{1'O^{-1}(1 - TOB)}.$$

Then, we obtain the following result related to the vacuity of the viability kernel and hence to question 1. The proof is given in Doyen, Rapaport & Terreaux (2001).

Proposition 1: Consider the total amount of resource $S > 0$. Under Assumptions A1 and A2, for any minimal harvesting level $\underline{h}$ greater than $\underline{h}^*(S,A)$, there does not exist an initial level resource x^0 and a sustainable policy from x^0.

We now deal with question 2 and attempt at computing the viability kernel $Viab_{\underline{h}}$ namely sustainable initial resource values. We consider an additional hypothesis on the matrix A:

Assumption A3: The matrix resource A is such that

i) $CA^n = CA^{n-1}$
ii) $CA^kB \le 0$, $k = 1, \dots, n-2$
iii) $CA^kB = 0$, $\forall k \ge n-1$

We obtain (the proof is given in Doyen, Rapaport & Terreaux (2001)):

Proposition 2: We posit assumptions A1, A2 and A3 on the resource matrix A and consider $\underline{h} > \underline{h}^*(S,A)$. Then the viability kernel is defined by

$$Viab_{\underline{h}} = \left\{ x \in \Re^n_+ \;/\; Ox \ge (1 - TOB)\underline{h},\ 1'x = S \right\}$$

4. EXAMPLE

We consider here the particular case where the matrix A represents an "eternal" population (i.e. without a natural mortality): $a_i = 1$ and $b_i = 0$, $\forall i$. This idealistic case can be seen as a modelling of a forest whose life expectation of trees is very large compared to the minimum age of cut and in which, under a "rational" exploitation, all trees will be renewed quite a long time before their age of natural death.

Assumptions A1 and A2 of Proposition 1 are fulfilled. The computation gives the largest sustainable value: $\underline{h}^*(S,A) = S/n$. Assumption A3 is also fulfilled. Then, Proposition 2 allows to characterize exactly the viability kernels:

$$\mathrm{Viab}_{\underline{h}} = \left\{ x \in \Re_+^n \,/\, Ox \geq \begin{pmatrix} 1 \\ 2 \\ \dots \\ n \end{pmatrix} \underline{h} \right\} = \left\{ x \in \Re_+^n \,/ \left\{ \begin{array}{c} x_n \geq \underline{h} \\ x_n + x_{n-1} \geq 2\underline{h} \\ \dots \\ \sum_{i=1}^{n} x_i \geq n\underline{h} \end{array} \right\} \right\}$$

Sustainable harvesting feedbacks h(x) are defined for any $x \in \mathrm{Viab}_{\underline{h}}$ by the set-membership (see Doyen, Rapaport & Terreaux (2001)):

$$h(x) \in \left[\underline{h}, \min_{i=1\dots n-1} \left(\sum_{i=1}^{n} x_i - (n-i)\underline{h} \right) \right] \tag{1}$$

We have considered for simulations a population structured in four layers with a total surface equal to S=10 units. Then, the largest sustainable value is: $\underline{h}^*(S,A) = 10/4 = 2.5$.

We have choosen for the simulations $\underline{h}=2$ as a desired sustainable value, and have simulated the trajectories provided by two kinds of selection of sustainable feedbacks:

1. The **maximal viable harvesting**, which consists in choosing for h(x) the largest value allowed by (1). We notice that this viable trajectory becomes cyclic (see Table 1).

t	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***
x_4	4	3	2	2	2	2	2	2	2
x_3	1	1	4	2	2	4	2	2	4
x_2	1	4	2	2	4	2	2	4	2
x_1	4	2	2	4	2	2	4	2	2
h_M	2	2	4	2	2	4	2	2	

TABLE 1: SIMULATION OF MAXIMAL VIABLE HARVESTING.

2. The **inertial viable harvesting**, which consists in choosing a feedback minimizing the change:

$$h_I(t,x) = \arg \min_{h \in [\underline{h}, h_M(x)]} |h(t-1) - h(t)|, \quad t \geq 1$$

We notice here that this viable trajectory becomes stationnary, contrary to the maximal sustainable harvesting (see Table 2).

t	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***
x_4	4	3	2	4	4	4	4	4	4
x_3	1	1	4	2	2	2	2	2	2
x_2	1	4	2	2	2	2	2	2	2
x_1	4	2	2	2	2	2	2	2	2
h_I	2	2	2	2	2	2	2	2	

TABLE 2: SIMULATION OF INERTIAL VIABLE HARVESTING.

5. CONCLUDING REMARKS

These results constitute a first step towards the construction of manegerial rules, allowing the foresters to adapt to the hazards and more generally to the (positive or negative) risks related to silviculture. Our goal was not to define an optimal trajectory, of which we would not have known a lot on its stability towards the change of the many parameters, but to give simple policies, depending on the state of the forest, so that it may fulfill in the future the demand of the society.

Therefore we are able to give some instantaneous and simultaneous "criteria" that materialize the "good health" of the bio-economic system, i.e. its viability.

Of course there remains a lot of work to be done in order to translate this viability method into practical rules. Moreover all the viable trajectories do not have the same interest since some of them will for instance lead to an overcapitalisation in old trees, which is not necessary good for the biodiversity, or from an economic point of view. But we are confident of the enrichment such an approach may bring to other prevailing management tool.

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