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EFITA 2001

MDPs for constrained management of renewable ressources

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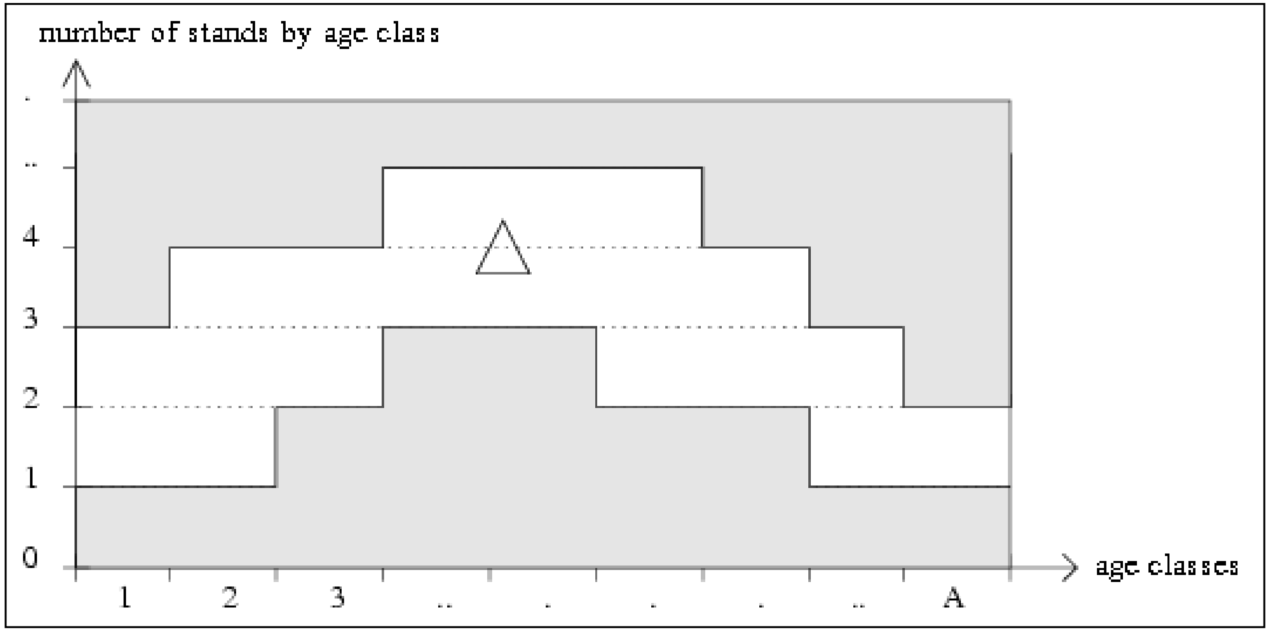
Purpose

- **Maintain bio-diversity in the forest, which goes through keeping a kind of homogeneity in the repartition of ages of the trees in the stands:**



Δ : set of satisfying age repartitions

2 approaches

- **To keep this homogeneity along the time, a first approach is to consider a classical MDP problem with a reward $r_t = 1$ if $s_t \in \Delta$ and 0 otherwise**

- **A second approach, based on solving a problem of optimization under constraints, is to use a Constrained MDP, the repartition of ages being an additional constraint to the maximisation of the timber sale revenue**

First approach: classical MDP

- **In this case, we consider at each time t a reward $r_t = 1$ if $s_t \in \Delta$ and 0 otherwise**

- **For the average reward function, our problem is then:**

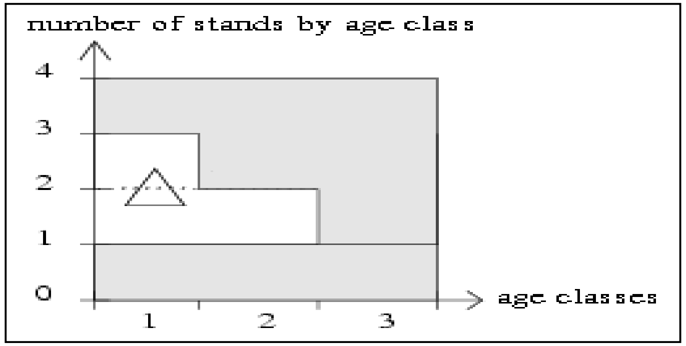
find $\pi^* \in \arg\max_{\pi} V^{\pi}$

with $V^{\pi}(s) = \begin{cases} \lim_{n\to\infty} \frac{1}{n} \sum_{t=0}^{n-1} E^{\pi}[r_t \mid s_0 = s] \quad \forall s \\ V^{\pi} \quad if \quad unichain \end{cases}$

$r_t = 1$ **if** $s_t \in \Delta$ **and 0 otherwise**

Example

- For example, for a forest of 4 stands, 3 age classes, 2 prevention levels, some given fire probabilities and the following Δ :



- We find an optimal deterministic markovien policy π^* with

$$V^{\pi^*}(s) = V^* = 0.8757 \qquad \forall s \quad (unichain)$$

$$\pi^* \quad defined \quad by$$

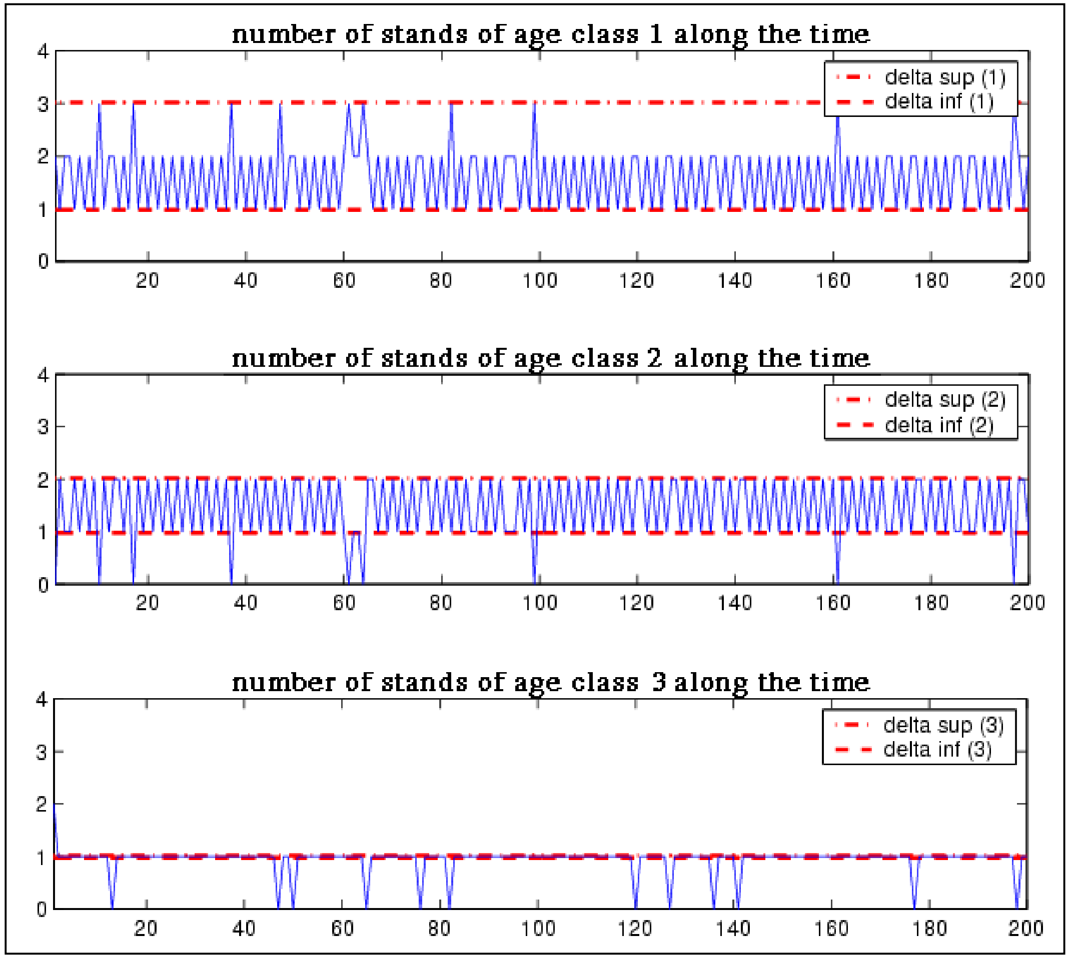
$$U^{\pi}(s) + V^{\pi} = \max_{a \in A}\left\{ r(s,a) + \sum_{s' \in S} p(s' \mid s,a) U^{\pi}(s') \right\} \forall s \in S$$

(Bellman's equation)

- So, the forest should be 87% of the time in a good repartition if we follow π^*

Example

- We can simulate the management for a given initial state to see if the optimal policy found is really effective:



- We stay in a good repartition most of the time (87% of the time)

Second approach: Constrained MDP

- **In this case, we consider at each time t a reward r_t of management of the forest (timber sales)**
- **We take into account the homogeneity of the forest as a constraint**
- **Our problem is now (also for the average reward function):**

find $\pi^* \in \arg\max_{\pi} V^{\pi}$ **subject to** $D^{\pi} \leq C$

with

$$V^{\pi}(s) = \lim_{n\to\infty} \frac{1}{n} \sum_{t=0}^{n-1} E^{\pi}[r_t \mid s_0 = s] = V^{\pi} \quad \forall s \quad (unichain)$$

$$r_t \quad (timber \quad sales)$$

$$D^{\pi}(s) = \lim_{n\to\infty} \frac{1}{n} \sum_{t=0}^{n-1} E^{\pi}[d_t \mid s_0 = s] = D^{\pi} \quad \forall s \quad (unichain)$$

$d_t = 0$ if $s_t \in \Delta$ and 1 otherwise

C Constraint value

Theory

- **Transformation in a Linear Program**

- **Approach based on occupation measures:**

$$\mu : \pi \to \mu(\pi)$$

$$V^{\pi} = \sum_{s \in S} \sum_{a \in A} \mu(s,a) r(s,a)$$

$$D^{\pi} = \sum_{s \in S} \sum_{a \in A} \mu(s,a) d(s,a)$$

NB: The optimal policy is here markovien stochastic

Linear Program

find $\pi^* \in \arg\max_{\pi} V^{\pi}$ **subject to** $D^{\pi} \leq C$

⇕

find $$\max_{\mu} \sum_{s \in S} \sum_{a \in A} \mu(s,a) r(s,a)$$

subject to $$\sum_{s \in S} \sum_{a \in A} \mu(s,a) = 1, \qquad \forall s,a \quad \mu(s,a) \geq 0$$

$$\sum_{a \in A} \mu(s',a) = \sum_{s \in S} \sum_{a \in A} \mu(s,a) p(s' | s,a), \quad \forall s' \in S$$

$$\sum_{s \in S} \sum_{a \in A} \mu(s,a) d(s,a) \leq C$$

our optimal policy π^* **then chooses action a at state s with probability:** $$\frac{\mu(s,a)}{\sum_{a' \in A} \mu(s,a')}$$

Example

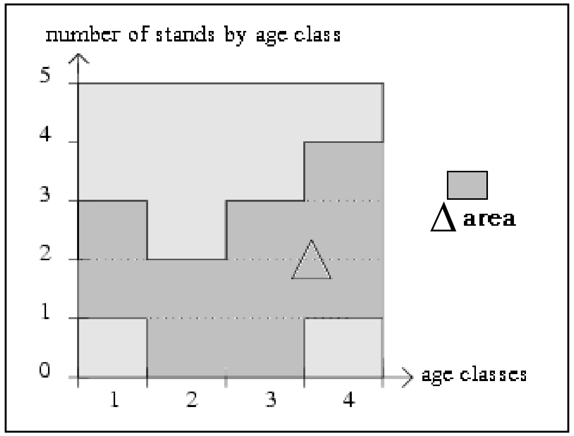
- In fact, the nature of the optimal policy depends on the value of C and Δ

- The optimal policy can be - deterministic
  - stochastic

  moreover, there can be no feasible policy for the problem

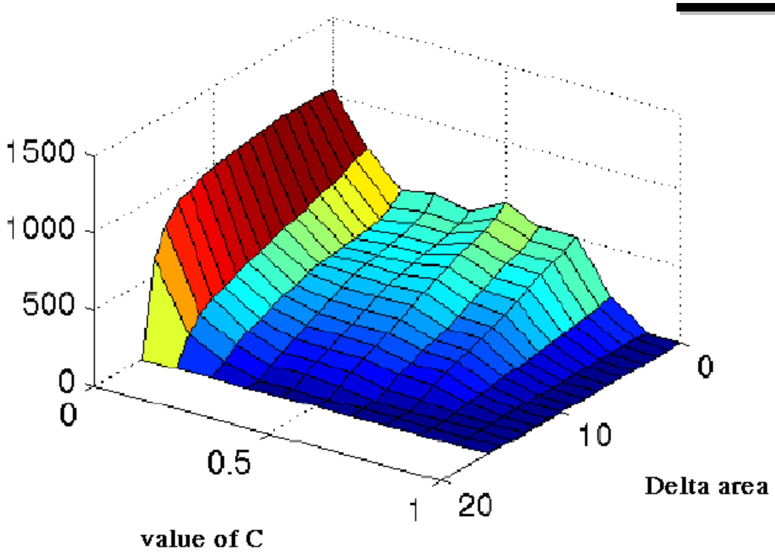
- For example, for a forest of 4 stands, 4 age classes, 2 prevention levels, some given rewards, some given fire probabilities, we can analyse the results for different values of C and Δ

- Δ area:

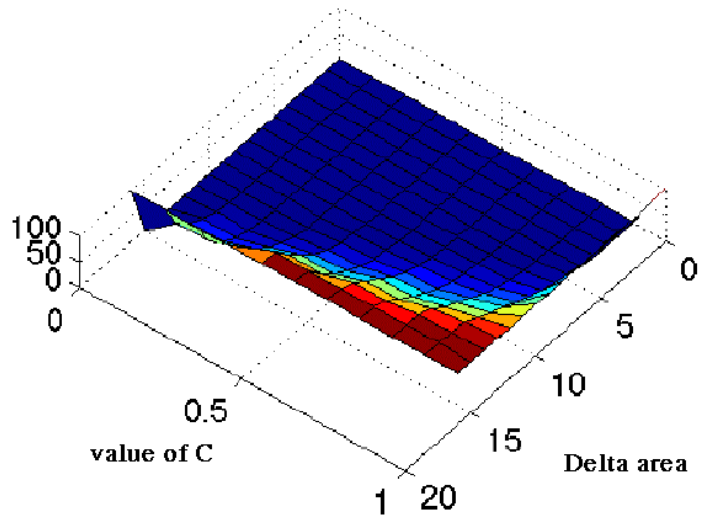


Example

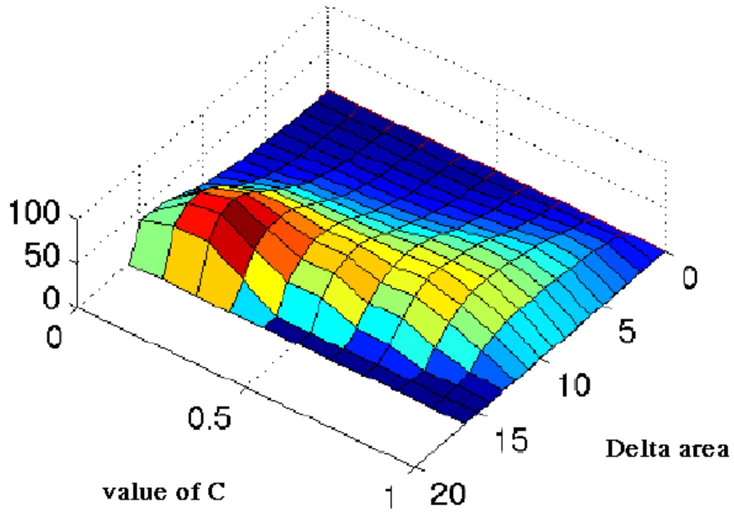
TOTAL NUMBER OF RANDOMIZATIONS



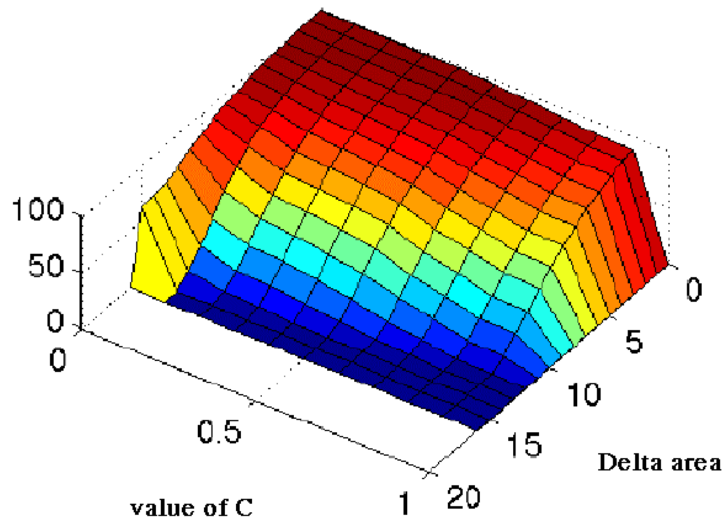
% OF DETERMINISTIC OPTIMAL POLICIES



% OF STOCHASTIC OPTIMAL POLICIES



% OF NON FEASIBILITY



Conclusion

MDP and CMDP: 2 approaches for constrained problems

- **MDP: “r=constraint (1 if s satisfying, 0 else)”**

  - easy to implement (known techniques)
  - does not take objective function optimization into account

- **CMDP: “r=timber sales & d=constraint”**

  - allows optimization while constraint satisfaction
  - may give stochastic optimal policies