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Dynamic production monitoring in pig herds

III. Modeling and monitoring mortality rate at herd level

Claudia Bono\textsuperscript{a}, Cécile Cornou\textsuperscript{a}, Søren Lundbye-Christensen\textsuperscript{b}, Anders Ringgaard Kristensen\textsuperscript{a,}\textsuperscript{∗}

\textsuperscript{a}HERD - Centre for Herd-oriented Education, Research and Development, Department of Large Animal Sciences, University of Copenhagen, Grønneårdsgade 2, 1870 Frederiksberg C, Denmark

\textsuperscript{b}Department of Cardiology, Cardiovascular Research Center, Aalborg Hospital, Aarhus University Hospital, Sdr.Skovvej 15, 9000 Aalborg, Denmark

Abstract

Management and monitoring systems may enable the farmers to enhance production results and reduce labor time. The aim of this paper is to develop a dynamic monitoring system for mortality rates of sows and piglets. For this purpose a model for mortality rates is implemented using a Dynamic Generalized Linear Model. Variance components are pre-estimated using an Expectation-Maximization algorithm applied on a dataset containing data from 15 herds, each of them including observations over a period ranging from three to nine years. Data are registrations of events for insemination, farrowing (including stillborn and live born), number of weaned piglets and death of sows. The model provides reliable forecasting on weekly basis. Detection of impaired mortality rate is performed by statistical control tools that give warnings when the mortality (rate) shows sudden or gradual changes. For each herd, mortality rate profile, analysis of model components over time and detection of alarms are computed for two categories, namely sows and piglets.

Keywords: Monitoring, Mortality model, Multivariate Dynamic Generalized Linear Model, Statistical control

1. Introduction

Mortality in pig farms is a considerable welfare issue, which also affects productivity results. Several studies have been performed on sow mortality (D’Allaire et al., 1991; Koketsu, 2000; Rueda Lopez, 2008) as well as on piglet mortality (Leenhouwers et al., 1999; Marchant et al., 2000; D’Allaire et al., 1991). The mortality rate is influenced by many factors as for instance the management systems, housing, environment, genotype, geography, feeding and climate (Duran, 2001). The factors that may influence the mortality rate in sows and piglets are often different.

Sow annual mortality rate may depend on the herd size, housing system and country (Duran, 2001). According to Sanz et al. (2007), it ranges from 7 to 17%. D’Allaire et al. (1991) estimated a rate of 14% whereas Chagnon et al. (1991) reported a range from 0 to 9.2%. Concerning the stages of mortality and according to Sanz et al. (2007), 60.6% of sows died during the insemination and gestation periods (around 17 weeks) and 39.4% during the lactation period (usually around 4 weeks), whereas Chagnon et al. (1991) reported that 42% of death cases occur in the peripartum period. Several studies (e.g. Koketsu, 2000; Duran, 2001) also report an effect of parity on the sow mortality rate.

For piglets, stillbirth rate ranges between 4.2 and 7.1%, whereas the pre-weaning mortality ranges from 10.8% to 13.2%

(1978; Marchant et al., 2000; Roehe and Kalm, 2000; KilBride et al., 2012). The total mortality rate, excluding mummified piglets, is around 18-20% (Marchant et al., 2000; Persdotter, 2010; KilBride et al., 2012), which is considered normal for the reproductive biology of the pig (Edwards, 2002). For the stillbirth rate a clear effect of parity has been reported (Leenhouwers et al., 1999; Canario et al., 2006; Weber et al., 2009) and several studies suggest that pre-weaning mortality is higher for first parity sows than for older sows (Fahmy et al., 1978; KilBride et al., 2012). Several studies have identified that the stage with the major mortality is the pre-weaning period (Roehe and Kalm, 2000; Wientjes et al., 2012). KilBride et al. (2012) stated that 84% of all pre-weaning mortality occur within the first 7 days of life.

Different methods have been used to estimate the mortality rates mentioned above. Several authors used simple Linear Models (Högborg and Rydhmer, 2000; Knol et al., 2002; Lund et al., 2002), others combined Linear Models and Threshold Linear Models (Grandinson et al., 2002; Arango et al., 2006), Roehe and Kalm (2000) used Generalized Linear Mixed Models.

Literature provides detailed studies concerning sow and piglet mortality. However no information is currently available on dynamic monitoring of mortality rate influenced by parity. In fact, a simple average over parities is not suitable for monitoring piglet and sow mortality because it depends on the age structure and the stage of the reproductive cycle. Nevertheless, the commercial systems currently used in Denmark only reports the mortality as simple quarterly averages over parities and stages. A new monitoring system for sow and piglet mor-
Mortality therefore needs to be developed in order to catch correlations between changes in sow and piglet mortality according to parity and the stage of the reproductive cycle, and to monitor changes over time.

If changes in mortality rates in one group (e.g. piglets) are correlated with changes in the other group (sows), the benefit of a combined model is that information about changes in one group will also influence the expectations for the other group. Since the forecast errors are used for monitoring, this is important. If the model is used for production forecasts it will be even more important. Therefore, all correlated mortality changes should be handled in the same aggregate model.

Recent improvements on dynamic monitoring have been done on litter size and on farrowing rate by Bono et al. (2012, 2013). Because mortality is, as farrowing, a binary trait, it is possible to follow the farrowing rate model suggested by Bono et al. (2013) with the appropriate amendments. To the authors’ knowledge, no application of Dynamic Generalized Linear Model (DGLM) on mortality rate has been reported.

The purpose of this paper is to develop a dynamic monitoring system for mortality rate in pig production. The mortality rate is modeled using a Dynamic Generalized Linear Model. Thereafter, statistical control tools are applied in order to detect systematic deviations, changes and other factors that may influence the mortality rate.

This paper is the third step of a larger project. The combination of the presented model with the previous two, on litter size and farrowing rate (Bono et al., 2012, 2013), represents a solid basis for the development of a new management tool that may be used to create a software able to dynamically monitor changes in the production process.

2. Explorative data analysis

Data used in the current study have been provided by the Danish Advisory Center. This dataset is the same as the one used in Bono et al. (2012, 2013). Data are registrations of events for insemination, farrowing (including stillborn and live born), number of weaned piglets and death of sows and weaners, obtained from 15 herds for a period ranging from three to nine years.

An explorative data analysis was performed on the dataset. Mortality rate has been calculated for both sow and piglet categories, for eight parities. Sow mortality has been furthermore divided into two groups: insemination and gestation periods, and nursing and dry periods. Piglet mortality has been analyzed for three groups: stillborn, pre-weaning and post-weaning mortality.

An overview of the mortality rates for the 15 herds is available in Table 1. Stillbirth rate ranged from 9.1 to 14 %, pre-weaning mortality is between 10.3 and 23.6 % whereas post-weaning mortality is quite low, ranging from 0.03 (suggesting registration failure) to 3.6 %. It should be noticed that post-weaning mortality data are not available for Herds 4 and 15 since these farms sell the piglets at weaning. For sows, the mortality in the gestation and insemination periods ranges from 1.4 to 3.2 % and in the nursing and dry periods from 1.8 to 5.5 % (the dry period here refers to the time from weaning to insemination). The reason for pooling the insemination and gestation periods as well as the nursing and dry periods is that it is neither possible in data to distinguish the insemination period from the gestation period nor the nursing period from the dry period. Only the farrowing dates and the insemination dates are known, so the cycle can only be split up into two stages.

Figure 1(a) illustrates the mortality rate for sows according to groups and parities. A sow mortality rate around 2% is observed for the insemination and gestation periods at first parity, and the maximum is reached at Parity 8 (around 4%). Higher values are observed in the nursing and dry periods, where the minimum rate is around 2.5% at Parity 1 and the maximum is around 5.5% at Parity 8. For piglets, as depicted in Figure 1(b), the stillbirth rate increases steadily with the parity number. The minimum rate is observed at Parity 1 (around 9%) whereas the maximum is around 18%, at Parity 8.

Figure 2 illustrates the correlation between groups: stillborn, pre-weaning mortality, post-weaning mortality, insemination and gestation mortality, and nursing and dry mortality. It is important to note that whereas sow and stillborn mortality can be computed parity wise, pre and post-weaning mortality cannot be grouped by parity due to the litter equalization (i.e. herd managers will often transfer some piglets from a large litter to another lactating sow which either has a smaller litter or has had her own biological piglets recently weaned). In this study, the number of pre-weaning piglets is indirectly calculated as the difference between weaned and live born piglets, at batch level (i.e. for all sows in a section). Visual inspection of Figure 2 indicates correlations between stillborn, pre-weaning mortality, insemination and gestation mortality, and nursing and dry mortality. No obvious correlation is found between post-weaning mortality and the other groups.

In general, it can be observed that sow mortality is highly dependent on parity with a peak in Parities 2 and 3, which is in accordance with D’Allaire et al. (1991) and Sanz et al. (2007); that sow mortality also depends on the stage of the reproductive cycle (Sanz et al., 2007; Chagnon et al., 1991); that stillbirth rate heavily depends on sow parity with an almost linear increase from Parity 1 to Parity 8 (Leenhouters et al., 1999); that pre-weaning mortality cannot be handled according to sow parity because of the litter equalization. Finally, it can be sug-
Table 1: Average mortality rate of the 15 herds according to categories and groups. The pre-weaning period is usually around four weeks and the post-weaning period around eight weeks.

<table>
<thead>
<tr>
<th>Herd</th>
<th>Piglets</th>
<th>Sows</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stillborn</td>
<td>Pre-weaning</td>
</tr>
<tr>
<td>1</td>
<td>11.1</td>
<td>13.4</td>
</tr>
<tr>
<td>2</td>
<td>12.2</td>
<td>15.4</td>
</tr>
<tr>
<td>3</td>
<td>11.3</td>
<td>13.6</td>
</tr>
<tr>
<td>4</td>
<td>11.4</td>
<td>16.5</td>
</tr>
<tr>
<td>5</td>
<td>10.1</td>
<td>11.1</td>
</tr>
<tr>
<td>6</td>
<td>13.2</td>
<td>14.4</td>
</tr>
<tr>
<td>7</td>
<td>14.0</td>
<td>23.6</td>
</tr>
<tr>
<td>8</td>
<td>12.5</td>
<td>12.8</td>
</tr>
<tr>
<td>9</td>
<td>10.6</td>
<td>11.0</td>
</tr>
<tr>
<td>10</td>
<td>13.4</td>
<td>18.4</td>
</tr>
<tr>
<td>11</td>
<td>12.2</td>
<td>14.5</td>
</tr>
<tr>
<td>12</td>
<td>10.3</td>
<td>10.3</td>
</tr>
<tr>
<td>13</td>
<td>13.0</td>
<td>14.9</td>
</tr>
<tr>
<td>14</td>
<td>12.1</td>
<td>13.1</td>
</tr>
<tr>
<td>15</td>
<td>9.1</td>
<td>16.5</td>
</tr>
</tbody>
</table>

Min | 9.1     | 10.3   | 0.03    | 1.4     | 1.8    |

Max | 14.0    | 23.6   | 3.6     | 3.2     | 5.5    |

Mean ± SD | 11.8 ± 1.4 | 14.6 ± 3.3 | 1.9 ± 1 | 2.3 ± 0.6 | 3.5 ± 1.2

Figure 2: Correlations between stillborn, pre-weaning mortality, post-weaning mortality, insemination and gestation mortality, nursing and dry mortality, for all herds.
gested that, due to correlations apparently observed between categories, mortality of sows and piglets until weaning should be handled in the same model.

3. The mortality model

Since mortality is a binary trait, it is natural to model the rate on the logistic scale and use the same multivariate binomial technique for a Dynamic Generalized Linear Model (DGLM) as described by Bono et al. (2013). The main reasons for using a DGLM instead of applying statistical control tools directly on the data are that the modeling approach enables us to give more precise forecasts and that the autocorrelation over time can be modeled directly, thus providing us with independent forecast errors.

We shall denote the mortality rate as \( p_G \) for pig group \( G \) and the corresponding logistic transform as \( \eta_G \) where

\[
\eta_G = \log \frac{p_G}{1-p_G}.
\]

A pig group, \( G \), is described by up to 3 identifiers. The first identifier is either \( s \) for sows, \( b \) for stillborn or \( p \) for piglets before weaning. The second identifier is the parity (if applicable) and the third is in stage in the reproductive cycle with the value 1 for the insemination/gestation period and 2 for the nursing/dry period. Thus, \( G = s32 \) refers to Parity 3 sows in the nursing/weaning period, \( G = b4 \) refers to piglets being born by a Parity 4 sow, and \( G = p \) refers to suckling piglets.

We first model the sow mortality, and notice that, due to the pattern seen in Figure 1(a), we can describe it by the following model for a sow in Parity \( n \), Stage \( j \) of the reproductive cycle

\[
\eta_{snj} = \mu + \alpha_n + \beta_j, \quad n = 1, \ldots, N, \quad j = 1, 2.
\]

Thus, the mortality is described as a general level, \( \mu \), adjusted for effect of parity, \( \alpha_n \), and effect of stage of the reproductive cycle \( \beta_j \). In order to ensure uniqueness of the estimates, we will assume in the following that \( \alpha_1 = \beta_1 = 0 \). Thus, \( \mu \) corresponds to mortality of Parity 1 sows in the insemination/gestation period. For stillborn piglets we use the following model for piglets born by a sow in Parity \( n \).

\[
\eta_{snj} = \mu + \alpha_n + \beta_j, \quad n = 1, \ldots, N, \quad j = 1, 2.
\]

In other words, we model the first 4 parities separately and assume a constant slope for parities higher than 4 as seen in Figure 1(b). Finally, the pre-weaning mortality will be described simply as

\[
\eta_p = \zeta
\]

where the \( \zeta \) value remains unchanged for all parities.

4. Sequential estimation technique

4.1. A multivariate dynamic generalized linear model

A multivariate dynamic generalized linear model consisting of an observation equation and a system equation will be applied. We will use weekly observations of dead sows and piglets to update the herd profile as described by Eqs. (2), (3) and (4). Since \( \alpha_1 = \beta_1 = 0 \), the latent parameter vector for week \( t \) will be

\[
\theta_t = (\mu, \alpha_{2n}, \ldots, \alpha_{2N}, \beta_{2n}, \gamma_1, \ldots, \gamma_4, \zeta, \delta_1)^T.
\]

4.1.1. Observation Equation

The observation vector \( Y_t \) consists of elements, \( y_{Gt} \), corresponding to all pig groups. The individual observation \( y_{Gt} \) is the number of dead sows or piglets in Group \( G \) out of \( N_{Gt} \) observed at week \( t \). Groups where \( N_{Gt} = 0 \) are left out of the observation vector.

The observation equations linking the observations to the parameters have the general form

\[
y_{Gt} \mid \theta_t \sim B(N_{Gt}, p_{Gt}),
\]

where \( B \) denotes the binomial distribution. The mortality rate \( p_{Gt} \) is equal to \( (\exp(-\eta_{Gt}) + 1)^{-1} \) (cf. Eq. (1)), and depends on the parameter vector \( \theta_t \) as follows:

\[
\eta_t = F_t \theta_t,
\]

where \( F_t \) is called the design matrix. The number of columns corresponds to the size of \( \theta_t \) and the number of rows corresponds to the number of non-zero values of \( \eta_t \).

In the observation of \( y_{Gt} \) and \( N_{Gt} \), a constant mortality for the entire stage is used according to the following principles for week \( t \):

- **Groups \( snj \), sows**: The number of dead sows for Group \( G = snj \) in week \( t \) is observed as \( y_{Gt} \). A list of sows present in the herd is maintained week by week. For each sow, the parity \( n \) and stage of reproductive cycle, \( j \), is maintained. Thus, \( N_{Gt} \) will be the number of sows (including those that die during the week) in parity \( n \), stage \( j \). In other words, a common weekly mortality rate is estimated for the entire stage.

- **Groups \( bp \), stillborn**: Here the observation for week \( t \) will be the number of total born piglets \( N_{Gt} \) and the number of stillborn piglets \( y_{Gt} \).

- **Group \( p \), suckling piglets**: In the database no information is available about dead piglets before weaning. The pre-weaning mortality is calculated indirectly as the difference between the number of weaned piglets and the number of live born. The observation for week \( t \) is the total number of live piglets at weaning \( N_W \) and the number of piglets originally born alive \( N_{Gt} \) by the sows having their piglets weaned in week \( t \). The number of dead piglets is then found as \( y_{Gt} = N_{Gt} - N_W \). The intensive use of the nursing sows in farms makes the weekly observation (arbitrary variation) full of noises. Therefore a period of four weeks observation period is suggested for pre-weaning piglets.

Now assume that, in week \( t \), \( N_{Gt} \) pigs are observed and \( y_{Gt} \) of them die. As an example for a herd, we assume for week \( t \) that
the observed values for a subset of the pig groups are as shown in Table 2. The observation vector contains an element for each group, where 

$$Y_t = (2,1,1,\ldots,0,65,\ldots,52,45,\ldots,12,71)',$$

and the design matrix will then look as follows (cf. Eqs. (2), (3) and (4)):

$$F = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}$$

where the vertical lines separate sections referring to different parameter groups of the vector \(\theta_t\). Thus, the leftmost section (first column) refers to the general herd level for sow mortality \((\mu_t)\). The next section (Columns 2-8) are coefficients referring to the parity effects on sow mortality \((\alpha_{2t}, \ldots, \alpha_{8t})\) and the third section (Column 9) refers to the effect of stage in the reproductive cycle \(\beta_{2t}\). The fourth and fifth section contain the coefficients for the parity specific stillbirth rate for Parities 1-4 \((\gamma_{1t}, \ldots, \gamma_{4t})\) in Columns 10-13, and in Column 14 the coefficient for the pre-weaning mortality \((\zeta_t)\). Finally the rightmost section (Column 15) holds the coefficient for the slope \(\delta_t\) of stillbirth rates after Parity 4 (i.e. the coefficient equals \(n - 4\) as in Eq. (3)).

4.1.2. System Equation

The system equation expresses how the parameter values may change over time. The general form of the system equation is

$$\theta_t = G_t \theta_{t-1} + w_t,$$

where \(G_t\) is called the system matrix, and \(w_t \sim N(0,W)\) where \(0\) is a vector of zeros and \(W\) is a variance-covariance matrix describing the evolution variance of each of the parameters (and the covariance), with free structure except for the slope of stillbirth rate after Parity 4 \(\delta_t\), which is set independent of the other variables. Since no particular systematic trend or pattern is expected, we assume that \(G_t = I\), where \(I\) is the identity matrix.

4.1.3. Weekly updating

The concept of univariate dynamic generalized linear models is well described in literature (e.g. West and Harrison, 1997; West et al., 1985). When it comes to multivariate binomial models the only application known to the authors is the previous work by Bono et al. (2013) where it was shown that the technique of univariate binomial models can be extended to also cover multivariate models. The developed updating technique is described in appendix A of Bono et al. (2013). It relies on Taylor expansion of the conditional probability function of \(y_{Gt}\) given \(I_{Gt}\). A key property of the technique is the fact that, for given \(I_{Gt}\), the observations \(y_{Gt}\) are independent. Using the described technique, it is possible to obtain weekly updated estimates for \(\theta_t\) and thus \(\eta_t\).

4.1.4. Initialization

In order to have a full specification of the DGLM, the initial information \(\theta_0 \sim N(m_0, C_0)\), i.e. before anything has been observed in the herd, must be defined. Because a binomial model is considered for mortality rate, the values are on the logistic scale. The initial means based on the results of the explorative data analysis are \(\mu_t: -4.03; \alpha_2\) to \(\alpha_8: 0.52, 0.48, 0.36, 0.38, 0.34, 0.52, 0.82; \beta_2: 0.36; \gamma_1\) to \(\gamma_4: -2.29, -2.25, -2.07, -1.90; \zeta: -1.77; \delta: 0.10.\)

It should be noted that the initial settings are of minor importance since the model will automatically adapt to the conditions of a specific herd. Therefore, for the variance-covariance matrix, we used a very rough approach assuming that the fifteen standard deviations correspond to an arbitrarily selected coefficient of variation of 40% and that the parameters are mutually independent.

4.1.5. EM-algorithm

The Expectation-Maximization (EM) algorithm technique is used in order to estimate the system variance \((W)\). The free software R (R Development Core Team, 2013) was used to compute the algorithms. The EM technique is an iterative algorithm based on Maximum Likelihood (ML) estimation and is described in details in Bono et al. (2012). In principle the implementation of the algorithm is straightforward, but in this case where the parameter vector has 15 elements, the variance-covariance matrix has 225 elements to estimate simultaneously. Even though the fact that a variance-covariance matrix is symmetric reduces the number of covariances to estimate to the half, it still turned out that the algorithm did not converge within the time frame available. A stepwise estimation technique was therefore applied.

The parameter vector \(\theta_t\) was partitioned into two sub-vectors, \(\theta_{S_t} = (\mu_t, \alpha_{2t}, \ldots, \alpha_{8t}, \beta_{2t}, \gamma_{1t}, \ldots, \gamma_{4t}, \zeta_t, \delta_t)'\) corresponding to sow mortality properties and \(\theta_{W_t} = (y_{1t}, y_{2t}, y_{3t}, y_{4t}, \delta_t)'\) corresponding to piglet mortality (including stillborn). Accordingly, the system error term \(w_t\) was partitioned as \(w_t = (w_{S_t}, w_{W_t})'\) and the system variance-covariance matrix as follows

$$W = \begin{pmatrix}
W_S & W_{SP} \\
W_{SP}' & W_P
\end{pmatrix}.$$
value of the \(i\)th row was then calculated from the conditional variance-covariance matrix using standard theory for multivariate normal distributions.

5. Detection by group of impaired mortality rate results

After the application of the DGLM, mortality rate of piglets and sows are monitored. In order to explain how the groups are filtered in the parameter vector, a practical example is reported.

Let us consider again the observation vector:

\[
Y_t = (2, 1, 1, 1, \ldots, 0, 65, \ldots, 52, 45, \ldots, 12, 71)'.
\]

In order to monitor only the sow mortality, the following filter vector

\[
\phi_s = (1, 1, 1, \ldots, 1, 1, 0, \ldots, 0, 0, \ldots, 0, 0),
\]

will be used. The total number of dead sows in week \(t\) is then \(\phi_s Y_t\). This allows to compute only sow specific results. If instead, the stillborn group is monitored, the following filter vector

\[
\phi_b = (0, 0, 0, 0, \ldots, 0, 1, 1, 1, 1, 1, 0),
\]

will be used. In this case, the filter vector allows monitoring of the number of stillbirth only.

For the last group, suckling piglets, the following filter vector

\[
\phi_p = (0, 0, 0, 0, \ldots, 0, 0, 0, \ldots, 0, 0, 1),
\]

will be used.

Let \(\phi\) be an arbitrary filter vector. The forecast for the total number in the group filtered out by the vector of deaths in week \(t\) is therefore \(\phi \mu_t\), with variance \(\phi \Sigma \phi'\) and the observed total number is \(\phi Y_t\). Thus the weekly forecast error, \(e_t\), is

\[
e_t = \phi Y_t - \phi \mu_t.
\]  

For each filtered group, the difference between the deviations of the observations and the predicted values, i.e. the forecast errors, are analyzed in a short and long time period.

For the short term period, control charts inspired by Shewhart (Montgomery, 2005) were used in order to detect alarms on a weekly basis. For the Shewhart Charts, the observation in week \(t\) is \(e_t\) and the standard deviation used for control limits, is

\[
s_t = \sqrt{\phi \Sigma \phi'}.
\]  

Thus, the numerical value of \(s_t\) will heavily depend on the number of deaths at week \(t\). Because \(\phi_s Y_t\) is a sum of several different binomial distributions with unknown value of the probability parameter, the distribution of the forecasted number of farrowings is not normal, so a standard Shewhart chart with symmetric control limits is not well suited. Due to the basically binomially distributed data with unknown probabilities, the control limits must be un-symmetric. In order to adapt the limits to this kind of data, a beta-binomial distribution was fitted in such a way that the mean and variance corresponded to the mean, \(\phi_s \mu_t\), and variance, \(\phi_s \Sigma \phi'\), of the forecast distribution. The lower control limit was defined as the 0.025 quantile of the beta-binomial distribution and the upper was defined as the 0.975 quantile. Both the limits were defined as integers (rounded down for the lower quantile and up for the upper) so that for each limit (upper and lower) there is a significance level that corresponds to approximately 2.5%. This approach corresponds exactly to what was done in Bono et al. (2013).

For the long term period, a V-mask was applied on the cumulative sum (Cusum) control chart in order to detect sudden or gradual changes of the mortality rate. These monitoring methods are described in details in Bono et al. (2012, 2013).

6. Results

Results of the system variance, of the model application and of the monitoring detection methods are shown in this section. The results are presented according to each category, namely

<table>
<thead>
<tr>
<th>Pig group</th>
<th>Group identifiers</th>
<th>Number at risk</th>
<th>Number of dead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sows, Par. 1, Ins.+Gest.</td>
<td>(s ) 1 1</td>
<td>150</td>
<td>2</td>
</tr>
<tr>
<td>Sows, Par. 1, Nursing+Dry</td>
<td>(s ) 1 2</td>
<td>75</td>
<td>1</td>
</tr>
<tr>
<td>Sows, Par. 2, Ins.+Gest.</td>
<td>(s ) 2 1</td>
<td>125</td>
<td>1</td>
</tr>
<tr>
<td>Stillborn, Par. 1</td>
<td>(b ) 1</td>
<td>621</td>
<td>65</td>
</tr>
<tr>
<td>Stillborn, Par. 4</td>
<td>(b ) 4</td>
<td>398</td>
<td>52</td>
</tr>
<tr>
<td>Stillborn, Par. 9</td>
<td>(b ) 9</td>
<td>56</td>
<td>12</td>
</tr>
<tr>
<td>Suckling piglets</td>
<td>(p )</td>
<td>654</td>
<td>71</td>
</tr>
</tbody>
</table>

Table 2: Example of mortality results according to pig group for sows and piglets in week \(t\) in a herd.
the sow and the piglet categories. All the 15 herds were included in the analysis.

6.1. System variance

The EM-algorithm was carried out according to the stepwise procedure described in Section 4.1.5. The two sub-matrices $W_1$ and $W_2$ were estimated independently and visual inspection of plots (not shown) indicated that both algorithms had converged after 5000 iterations. The row-wise estimation of the covariance matrix $W_{SP}$ converged almost instantly.

The values of the full variance-covariance matrix are presented in Table 3. Variance of the following components is presented in Table 3. Variance of the following components is presented in the diagonal: sow mortality ($w_1$), parity effects ($w_2$ to $w_6$), effect of stage ($w_7$), stillbirth rate for the 4 first parities ($w_{10}$ to $w_{13}$), pre-weaning mortality ($w_{14}$) and slope for stillbirth after Parity 4 ($w_{15}$). Correlations between all the parameters are shown below the diagonal.

As it is seen from the lower left partition of the matrix, the correlations between fluctuations in sow mortality ($w_1, \ldots, w_6$) and piglet mortality ($w_{10}, \ldots, w_{15}$) are very small. On the other hand, the correlation within group (sows or piglets) are often very high confirming that if mortality increases for one parity it tends to increase almost identically for other parities. For sows, this effect is to some extent hidden by the fact that $w_1$ corresponds to the general level, whereas the parity effects are measured as deviations. If the model is parameterized so that for the first four parities the vector $\phi_1 = (\phi_{11}, \ldots, \phi_{40})'$ reflects the absolute parity specific mortality (on the logistic scale), we have $\phi_1 = A(\mu_1, \alpha_2, \alpha_3, \alpha_4)'$ where $A$ is a matrix having ones in the diagonal and in the first column and zeros elsewhere. In this parameter space the system equation would be $\phi_t = \phi_{t-1} + \omega_t$, where $\omega_t \sim N(0, AW_t A')$ with $W_t$ being the upper left $4 \times 4$ partition of $W$. Numerically, we get for the correlation matrix corresponding to $AW_t A'$

$$\text{Corr}(\omega_t) = \begin{pmatrix} 1.000 & 0.961 & 0.904 & 0.853 \\ 0.961 & 1.000 & 0.875 & 0.799 \\ 0.904 & 0.875 & 1.000 & 0.981 \\ 0.853 & 0.799 & 0.981 & 1.000 \end{pmatrix}$$

thus confirming very high correlations between mortality changes of different parities.

6.2. Model components

Figures 3 and 4 show a detailed analysis of the DGLM components for Herd 10, over three years. Smoothed means for the first five parities are shown for sows in the two stages. In both stages, Parity 2 has a higher weekly mortality rate, but in general the parity differences are small. The weekly mortality rate is in general slightly lower in Stage 1 - particularly in the beginning. The evolution of the smoothed means over time indicates that the first five parities follow a similar pattern.

Concerning piglets, the smoothed mean of the stillbirth rate for four parities and the smoothed pre-weaning rate are shown in Figure 4. As compared to sow results, the stillbirth rates for the first four parities (Figure 4(a)) fluctuate far more due to the much higher system variance for the stillbirth property. Parities 1 and 2 have a weekly mortality rate that fluctuates between 0.07 and 0.14, whereas Parities 3 and 4 have a higher weekly mortality rate ranging from 0.09 to 0.17. Pre-weaning piglet mortality (Figure 4(b)) ranges between 0.16 to 0.25.

6.3. Detection of alarms in mortality rate

This section illustrates possible results of using the control charts on real data. Since, however, no additional information about the herds is available, it is not possible to distinguish false alarms from true alarms where something has actually happened in the herd.

Monitoring methods were applied separately for sow and piglet mortality. Four different filter vectors (Section 5) were applied in order to obtain the alarms for the following groups:

1. Sows,
2. Stillbirths,
3. Pre-weaning piglets,
4. All piglets.

The Sow group combines mortality rate of both Stage 1 and 2, and all piglets combines stillbirth and pre-weaning mortality rates.

Results for sow and stillbirth groups are illustrated for two different time spans: short (26 weeks) for the weekly control, and long (156 weeks) for the long term control. The use of a control chart for weekly monitoring of the mortality rate is illustrated in Figure 5(a) for sows and in Figure 6(a) for piglets. The central line (black plain line) represents the difference between observed and predicted values. The dotted lines are the control limits, which are asymmetric due to the nature of the mortality.
Table 3: Estimated system variance-covariance. Values in and above the diagonal have been multiplied by $10^3$. Values of the correlations are shown below the diagonal. The partitioning reflects the estimation procedure where first the upper left and lower right sub-matrices were estimated independently and afterwards the covariances of the upper right and lower left sub-matrices were estimated row by row.

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<th>Piglet mortality (including stillbirth)</th>
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The long term monitoring, which corresponds to 0.2 alarm per herd per year, have been found for sow mortality in the short period. The average of 1.1 alarms per herd per year. No decreasing alarms into account, the average is 0.5 alarm per herd per year. For the increasing alarms, i.e. when the mortality rate significantly increases, are taken into account, the mean per herd per year becomes 1.9.

Finally, when the mortality rates for all piglets are combined and detected as a whole, 19 alarms are observed using the V-mask (0.4 per herd per year), of which 9 of them are increasing (0.2 per herd and per year). A total of 161 alarms were triggered using control charts (3.6 per herd per year, or 1.9 for increasing alarms).

7. Discussion

Only very few previous studies have dealt with dynamic monitoring of mortality in pig herds. An older study has previously monitored stillborn using control charts. Wrathall (1977), reported in de Vries and Reneau (2009), discussed methods of control to monitor, among other things, fetuses born dead. Later the same author (Wrathall and Hebert, 1982) also used control charts but focused on live born per litter. Also focusing only on live born in total, Krieter et al. (2009) applied cumulative sum (CUSUM) control chart and exponentially weighted moving average (EWMA) to detect small deviations in production process of simulated sow herd datasets. Monitoring mortality using Shewhart control charts has only been performed for finisher pigs in Fraile et al. (2009), where the authors assess changes in performance before and after the addition of treatment in the feed.

While the previous attempts mentioned have used statistical control tools directly on data, the approach in this study has been first to apply a DGLM on the data and then afterwards to use statistical control tools on the residuals. There are basically two reasons for including the DGLM step in the analysis. One of the properties of such models is that when an explicit model is applied on observations through the system and observation equations, making forecasts become easier. Hence, it is possible to obtain better expected values. Another advantage of dynamic linear models is the dependence of the parameters over time. In fact, in animal production, there are often repeated measurements of the same animals or similar environmental effects. Therefore, it is possible to estimate the autocorrelation. Once the autocorrelation has been identified it is possible to model it and to obtain the forecast errors which, assuming that the autocorrelation model is correct, are independent and therefore fulfill the requirements of the control charts applied afterwards.

In other words, better expected values and dependence over time are the main reasons for using a DGLM as part of the dynamic monitoring in this study.

In the explorative analysis of this study, correlations between stillborn, pre-weaning and post-weaning mortality, insemination and gestation mortality, and nursing and dry mortality, have been investigated (Figure 2). On this basis, the model was defined to simultaneously analyze mortality rate of sows and piglets even though the potential correlations traced in the figure refer to the mortality levels of different pig groups in a herd. What is modeled by the combined mortality model presented in

![Figure 5: Monitoring methods applied for Herd 10 for sow mortality. (a) Weekly monitoring (period of 26 weeks). Control Chart where the central line represents the observed - predicted observations, and the dotted lines are the upper and lower control limits. (b) Long monitoring period (period of 156 weeks).](image)

![Figure 6: Monitoring methods applied for Herd 10 for stillbirth. (a) Weekly monitoring (period of 26 weeks). Control Chart where the central line represents the observed - predicted observations, and the dotted lines are the upper and lower control limits. (b) Long monitoring period (period of 156 weeks).](image)
Based on the estimated variance-covariance matrix (Table 3) it is concluded that such a hypothesis cannot be confirmed since the correlations between changes in parameters referring to sows and parameters referring to piglets are close to zero. In fact they are all below 0.05 implying that for any practical considerations the two parameter groups are independent. It can therefore be concluded that there is no benefit from modeling sow and piglet mortality in the same model. The two groups can without loss of information be handled separately which would also ease the estimation of variance components by the EM algorithm considerably.

From the variance components of Table 3 it is also clear that the system variances for sow mortality are very small and highly mutually correlated. In other words, the pattern of sow mortality seems to be a very stable property within herd as it is also illustrated by Figures 3. This property makes the developed method well suited for monitoring sow mortality since a well-established pattern makes it easier to detect when and if the pattern changes.

For piglets (stillborn as well as those dying in the nursing period) the system variances are much higher and only loosely mutually correlated. This is very clear when comparing the piglet plots of Figure 4 to the sow plots of Figure 3. In fact the piglet results fluctuate so much that they hardly make sense for monitoring. For a better graphical evaluation a solution could be to reduce the system variance (only for plotting purposes) in order to produce more stable plots.

One of the reasons for the high system variance of piglet mortality is probably inaccuracies in the observation procedure. In practice it is not always obvious whether a dead piglet found in the farrowing pen is stillborn or it is dead after birth. It may therefore often happen that they are misclassified and thus contributing to the variation. Particularly for the pre-weaning mortality there is also the problem that the number of dead piglets is calculated indirectly as the difference between liveborn and weaned piglets. As previously mentioned, the intensive use of nursing sows makes it difficult to relate the piglets being weaned in a given week to a specific birth week.

Given the high system variances for piglet mortality it is concluded that the proposed model is less well suited for monitoring piglet mortality. However, improved registration methods for piglets in the future may very well change this conclusion.

Detection methods were applied to monitor changes in a short and long time-span. Results of the total number of alarms were presented in Table 4, where four filters were applied to obtain the total number of alarms for four different groups. The percentage of alarms (increasing and decreasing) for the control charts was, in average, around 2% for sows and around 7% for piglets. Given the settings of the control chart (attempted 95% confidence limits), at least 5% alarms should be expected. It is, therefore, initially surprising that the number of alarms for sows is only around 2%. It should, however, be recalled that with binomially distributed data the observations only take integer values and since the expected number of dead sows in a given week is very low, rounding errors may play a big role in the result, since the upper confidence limit is set to the smallest integer above the 97.5 percentile (which is typically a higher percentile). Furthermore, the lower confidence limit will almost always be zero deaths, implying that no “alarm” can ever be triggered.

The percentage of alarms from the V-mask was, in average, around 1% for sows and 0.8% for piglets. Rounding up the integer values used in the control limits would have resulted in more alarms, closer to the expected 5% (with 95% confidence interval). It must, however, be emphasised that the settings of the V-mask are chosen rather arbitrarily in this study. In case of implementation for use in practise, a calibration of the settings under known production circumstances needs to be performed.

### 8. Conclusion

A system for simultaneously monitoring of mortality rate for sows and piglets was developed. It is based on a Dynamic Generalized Linear Model, with weekly updates, combined with monitoring methods for short (weekly) and long term periods. The weekly evolution of sow mortality turned out to be independent of the corresponding evolution for piglets. It can therefore be concluded that there is no benefit from modeling sow and piglet mortality in the same model. The two groups can without loss of information be handled separately.

The model has shown to work properly particularly for sows where the system variances are very small and highly mutually correlated. For piglets, where much higher system variances were computed, the proposed model is less well suited for monitoring piglet mortality. However, improved registration methods for piglets in the future may very well change this conclusion.
The combination of this model with the previous ones (Bono et al., 2012, 2013), will help developing a management tool to help the farmers to monitor production, make decision, prevent problems, and reduce economical losses.

Conflict of interest statement

The authors report that there is no conflict of interest relevant to this publication.

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