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Final preprint (accepted version) of article published in Livestock Science. Please cite as:


DOI: 10.1016/j.livsci.2012.01.003
Optimal slaughter pig marketing with emphasis on information from on-line live weight assessment

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Abstract

One of the most labor intensive tasks of a traditional slaughter pig production is weighing of pigs for marketing, and in several recent projects methods for automatic assessment of live weights are developed. For an optimal utilization of the resulting weighing data, a decision support system for optimization of the marketing policy is needed. In this paper, a pen level model intended as the core of such a decision support system for optimization of slaughter pig marketing is presented. The model is based on a hierarchical Markov process and emphasis is put on definition of the state space in such a way that the observations from the online live weight estimation may serve as input to the decision support system and thus improve the precision of the underlying predictions of future growth through a learning algorithm based on Kalman filtering in a Dynamic Linear Model. The aim is to become able to inform the farmer on the number of pigs being ready for marketing in each individual pen. The MLHMP software system is used for implementation of the model.

Keywords: Replacement, Dynamic programming, Markov decision process, Learning, Dynamic Linear Model, Pig, Slaughter weight

1. Introduction

There is a growing need for computer-based methods for management support in pig production. Simultaneously, the possibilities for real-time monitoring of the production have improved as a consequence of modern computer technology and development of improved statistical methods. Average herd size is increasing and in the construction of new production systems one of the main objectives has been to reduce the input of human labour. The number of animals managed per person is therefore also increasing. One of the most labour intensive tasks of a traditional slaughter pig production is weighing of pigs for marketing, and in several projects methods for automatic assessment of live weights through image analysis or other automated methods are developed (Rydberg and Gilbertsson, 2004; Jørgensen, 2007; Schofield, 2007). When the equipment is installed in a production system, online estimation of live weights of the pigs is available throughout the fattening period. If those estimates are processed in an appropriate way, they serve as valuable input for prediction of growth as well. They may therefore be used as observations for a decision support tool for optimal marketing of the slaughter pigs.

The problem of optimal slaughter pig marketing can be regarded as a sequential decision problem involving decisions at two different levels: the animal level and the batch level. At the animal level decisions comprise of selecting and marketing of individual pigs based on some kind of observation of live weight, and at batch level the decision is when to terminate the batch (market the remainder of the batch and insert a new batch of weaners). At batch level, the decision is based partially on observations of the number of remaining pigs, their live weight distribution and expected growth and partially on the operational constraints of the slaughter pig unit, where the most important one probably is the weaner supply. Aspects of the problem have been dealt with before (Jørgensen, 1993; Kure, 1997; Toft et al., 2005; Niemi, 2006; Boys et al., 2007; Ohlmann and Jones, 2008), but the online live weight assessment is a new aspect which will be included in the present study.

A pen level model for optimization of slaughter pig marketing is presented. It is based on a hierarchical Markov process and emphasis is put on definition of the state space in such a way that the observations from the online live weight estimation may serve as input to the decision support system and thus improve the precision of the underlying predictions of future growth by means of learning algorithms based on Kalman filtering in a Dynamic Linear Model (DLM). The aim is to become able to inform the farmer on the number of pigs being ready for marketing in each individual pen. The MLHMP software system (Kristensen, 2003) has been used for implementation of the model.

2. The monitoring system and the price system

The model is implemented independently of the online weight assessment system. It may be based on any principle (e.g. image analysis or mechanical weighing). We shall just
make some assumptions concerning the output provided by the system. The basic assumption is that if a pen with \( n \) pigs is monitored, the system is able to supply \( n \) weight estimates

\[
\hat{w}_i(t) = w_i(t) + \epsilon_i,
\]

where \( \hat{w}_i(t) \) is the observed weight of the \( i \)th pig at time \( t \), \( w_i(t) \) is the true live weight, and \( \epsilon_i \sim N(0, \sigma^2) \) is the measurement error, where we assume that the precision (i.e. \( 1/\sigma^2 \)) is known. It must be emphasized that the pigs are not identified. We only know that the observed weight distribution is given by \( \hat{w}_1, \ldots, \hat{w}_n \). We are not able to identify a particular weight estimate with a specific pig. Note, that through the weight estimates we are also able to answer questions of the type: “How many pigs in the pen have an estimated weight higher than \( \delta \)?” This is important for the implementation of a delivery policy based on threshold weights as described later.

In Denmark the price obtained for a slaughter pig depends on the slaughter weight, where the highest price per kg is obtained in a range around 77 kg. For higher as well as lower slaughter weight the price is reduced proportionally with the deviation from the optimal interval. Furthermore, there is a price reduction for lean meat percentages below 61 and a premium for slaughter weights in a range around 77 kg. For higher as well as lower slaughter weight the price is reduced proportionally with the deviation from the optimal interval. If we, for instance, assume weekly intervals, we have

\[
e(t) = \alpha e(t-1) + \eta_t
\]

where \( \alpha \) is an auto regression coefficient, and \( \eta_t \sim N(0, \sigma^2_\eta) \) is an independent random term. Eq. (4) will be one of the system equations of the DLM.

As described so far we have only used the average observed weight as a source of information. Since we have actually observed \( \hat{w}_1, \ldots, \hat{w}_n \) we have much more information available about the distribution. We could for instance calculate the sample variance, but the problem is that the distribution of the sample variance is not normal. It is therefore difficult to include it in a DLM. Instead we shall use the 0.16 sample quantile \( \hat{w}_{(0.16)} \) as a further description of the underlying distribution of live weights. Since the distribution of a sample quantile is asymptotically normal around the true quantile, we have

\[
\hat{w}_{(0.16)}(t) = \overline{y}(t)L + e(t) - \rho(t) + \tau_t
\]

where \( \rho(t) \) is the standard deviation of \( \hat{w}_i(t) \), and \( \tau_t \sim N(0, \sigma^2_\tau) \) expresses the sample uncertainty. In Eq. (5) we use the well known fact that in a normal distribution, the 0.16 quantile is the mean minus the standard deviation. With a rather limited number of pigs in a pen, the exact 0.16 quantile is difficult to observe, so in practice the \( n \) observed weights are sorted, and the kth order statistic is selected as the observation, where \( k \) is the integer making the fraction \((k-1)/n\) “close to” 0.16. Since, it is not likely, that \((k-1)/n\) exactly 0.16, \( \rho(t) \) must be multiplied by an adjusting factor \( \alpha \) close to 1 in Eq. (5). The factor only depends on the values of \( n \) and \( k \), and it is easily determined from the properties of a normal distribution. If, for instance \( n = 20 \) and \( k = 3 \) (indicating that the third from bottom of 20 observations is used), the value becomes \( \alpha = 1.13 \). Thus, the final version of Eq. (5) becomes

\[
\hat{w}_{(0.16)}(t) = \overline{y}(t)L + e(t) - q(t) + \tau_t
\]

where \( \hat{w}_{(0.16)}(t) \) denotes the \( k \)th order statistic of the observations.

As concerns the development of \( \rho(t) \) (the standard deviation) over time, we assume that it increases linearly over time (as the pigs grow). In case of linear growth of the pigs, this corresponds to constant coefficient of variation. This leads to the following relation:

\[
\rho(t) = \frac{t}{t-1} \rho(t-1).
\]
We are now ready to specify the full DLM. The two-dimensional observation vector is \( Y_t = (\tilde{\eta}(t), \tilde{\epsilon}(t, t \neq 0)) \) and the parameter vector is \( \theta_t = (L, e(t), \rho(t)) \). The observation equation is
\[
\begin{pmatrix}
    \tilde{\eta}(t) \\
    \tilde{\epsilon}(t, t \neq 0)
\end{pmatrix} =
\begin{pmatrix}
    1 & 0 \\
    0 & 1 - \alpha
\end{pmatrix}
\begin{pmatrix}
    L \\
    e(t)
\end{pmatrix} +
\begin{pmatrix}
    \xi_t \\
    \eta_t
\end{pmatrix}
\tag{8}
\]
or in short notation
\[
Y_t = F_t \theta_t + \zeta_t
\tag{9}
\]
where the matrix \( F_t \) and the random vector \( \zeta_t \sim N(0, \Xi) \) are easily identified by comparing Eqs. (8) and (9). The system equation of the DLM is
\[
\begin{pmatrix}
    L \\
    e(t) \\
    \rho(t)
\end{pmatrix} =
\begin{pmatrix}
    1 & 0 & 0 \\
    0 & \alpha & 0 \\
    0 & 0 & \frac{1}{1 - \alpha}
\end{pmatrix}
\begin{pmatrix}
    L \\
    e(t - 1) \\
    \rho(t - 1)
\end{pmatrix} +
\begin{pmatrix}
    0 \\
    0 \\
    0
\end{pmatrix}
\tag{10}
\]
or in short notation
\[
\theta_t = G_t \theta_{t-1} + \xi_t
\tag{11}
\]
where the matrix \( G_t \) and the random vector \( \xi_t \sim N(0, \Xi) \) are identified by comparing Eqs. (10) and (11). The matrix \( G_t \) simply implies that the expected value of \( L \) is assumed to stay constant, that the expected value of the sample uncertainty \( e(t) \) is \( a e(t - 1) \) as also said in Eq. (4) and that the standard deviation develops as described by Eq. (7).

In addition to the observation and system equations we also need to specify the initial distribution of the parameter vector \( \theta_0 \sim N(\hat{\theta}_0, C_0) \). In other words, we must specify an initial mean vector \( \hat{\theta}_0 \) and an initial variance covariance matrix \( C_0 \).

Having defined the DLM, we may at regular intervals use the Kalman filter as described by West and Harrison (1997) for updating of the distribution of the parameter vector \( \theta_t \sim N(\hat{\theta}_t, C_t) \). The updated mean value \( \hat{\theta}_{t+1} \) (which we consider as the updated estimate for the parameter vector) is calculated as
\[
\hat{\theta}_{t+1} = G_{t+1} \hat{\theta}_t + A_{t+1} d_{t+1},
\tag{12}
\]
where
\[
d_{t+1} = Y_{t+1} - F'_{t+1} G_{t+1} \hat{\theta}_t
\tag{13}
\]
and
\[
A_{t+1} = R_{t+1} F_{t+1} Q_{t+1}^{-1}
\tag{14}
\]
The matrix \( R_{t+1} \) is the prior variance of \( \theta_{t+1} \) (before observing \( Y_{t+1} \)), and the matrix \( Q_{t+1} \) is the variance-covariance matrix of a one-step forecast of \( Y_{t+1} \). We have
\[
R_{t+1} = G_{t+1} C_{t} G'_{t+1} + \Xi
\tag{15}
\]
and
\[
Q_{t+1} = F'_{t+1} R_{t+1} F_{t+1} + Z.
\tag{16}
\]
Finally, the updated variance-covariance matrix of the parameter vector \( \theta_{t+1} \) is calculated as
\[
C_{t+1} = R_{t+1} - A_{t+1} Q_{t+1} A'_{t+1}
\tag{17}
\]

The updated mean vector \( \hat{\theta}_{t+1} \) will depend on the observed values whereas the updated variance covariance matrix \( C_t \) will be independent of the observed values (but it will depend on the number of observations).

For calculation of the transition probabilities later in the optimization model, it should be noticed that
\[
(\hat{\theta}_{t+1} | \hat{\theta}_t) \sim N(G_{t+1} \hat{\theta}_t, A_{t+1} Q_{t+1} A'_{t+1})
\tag{18}
\]
as also mentioned by Nielsen et al. (2011).

### 3.2. Slaughter weight and lean meat percentage

In Denmark, slaughter pigs are priced according to slaughter weight and lean meat percentage. From a management point of view this is a problem, because the farmer is only able to observe live weight and, indirectly, daily gain during the fattening period. In order to be able to predict the price of a pig delivered to slaughter a model for prediction of slaughter weight and lean meat percentage given live weight and daily gain is needed.

For conversion of live weight, \( w_t \), to slaughter weight, \( w_s \), the following model is used:
\[
w_s = c w_t + e_s,
\tag{19}
\]
where \( c \) is here regarded as a constant and \( e_s \) is a normally distributed random term distributed as \( N(0, \sigma^2) \).

As concerns the lean meat percentage, there is an interaction with the growth rate. Based on several experiments with ad libitum or restricted fed pigs, the Danish Pig Research Centre, as a rule of thumb, estimates the effect of feeding a pig 0.1 FEsv\(^1\) extra per day (Pedersen, 2001) to be i) an increased daily gain of 40 g, ii) an impaired feed conversion rate of 0.004 FEsv extra energy per kg gain, and iii) an impaired lean meat percentage of 0.3.

So if age at slaughter is increased due to a lower intake of the normal diet the lean meat percentage will increase. This could be the case if there is a difference in growth rate in ad libitum fed slaughter pigs, if the difference is caused by lower intake by some of the pigs due to less appetite or low social ranking, which could cause restricted access to the feeder. If the age is increased due to some sort of illness or digestive malfunction the lean percentage may not necessarily be increased.

If on the other hand some pigs grow faster than average it is assumed that they will have a lower lean meat percent.

Pigs fed restricted (e.g. liquid feeding systems) will also have an increased lean meat percent and slaughter age and a better feed conversion ratio.

The relation between growth rate, lean meat percent and feed conversion ratio varies widely between herds. Therefore

\(^1\)FEsv is the Danish energy unit used in pig feeding. 1 FEsv equals 7.72 MJ net energy.
it is necessary to base the calculations of the connection between lean meat percent (and perhaps feed conversion ratio) and slaughter age on production results from the specific herd. Based on the mentioned rule of thumb the lean meat percentage, \( \lambda_j \), of a specific pig, \( j \), can be estimated from the herd’s average lean meat percentage, \( \bar{\lambda} \), and the pig’s deviation in growth rate from the herd average by the equation

\[
\lambda_j = \frac{-0.3(g_j - \bar{g})}{40} + \bar{\lambda},
\]

where \( g_j \) is the daily gain (g) of the individual pig, and \( \bar{g} \) is the herd average. Based on the number of days since the pigs entered the pen, the transfer weight and the current live weight, the expected lean meat percentage of an individual pig may be calculated according to Eq. (20).

### 3.3. Feed intake

The feed intake is modelled as the sum of feed for maintenance and feed for growth. The basic relation between daily energy intake, \( f \) (FEsv), live weight, \( w_l \), and daily gain, \( g \), is

\[
f = k_1 g + k_2 w_l^{0.75},
\]

where \( k_1 \) and \( k_2 \) are constants describing the use of feed per kg gain and per kg metabolic weight, respectively. The two constants have in the general case been estimated by Jørgensen (2003) corresponding to “average” feed conversion rate. Thus, \( k_1 \) was estimated to 1.549 FEsv per kg gain, and \( k_2 \) to 0.044 FEsv per kg metabolic weight per day. The feed conversion rate is very important for the cost of pig production. At farm level the feed conversion rate is measured as FEsv per kg gain, as an average over the observed growth interval (30-100 kg). In the model, the feed conversion rate is split into a part for maintenance and a part for growth as it appears from Eq. (21). This enables a better calculation of the costs of keeping a pig another week, because the amount of feed used for maintenance will increase as the pig grows.

### 4. Optimization of delivery policies

An optimization model based on a multi-level hierarchical Markov process (Kristensen and Jørgensen, 2000) was built. For examples of working models of this type in other application areas, reference is made to Lien et al. (2003), Nielsen et al. (2004), Kristensen and Søllested (2004a,b), Ge et al. (2010), Nielsen et al. (2010) or Rodríguez et al. (2011).

In this section, we shall give a description of the model. The system being modeled is a pen with a given capacity expressed by the number of pigs, \( n_0 \), it may contain. It is assumed that the pen must be completely emptied before the new batch is inserted. Delivery of pigs will occur in a fixed number of possible deliveries starting a predetermined number of weeks (e.g. 4) before termination. After the first potential delivery, pigs are selected and delivered once a week.

Two scenarios concerning weaner supply are considered:

**Flexible:** New piglets are bought and inserted as soon as the pen is empty (and has been cleaned).

**Constant:** New piglets are bought and inserted at regular predefined time intervals (e.g. 10 weeks).

#### 4.1. Founder level

The founder process is an infinite stage Markov decision process where each stage corresponds to a group of pigs occupying the pen. In this first version of the model it is assumed that successive batches are independently sampled from the same distribution concerning relative growth capacity \( L \). Thus, the initial estimate, \( L_0 \) for \( L \) is the population mean (i.e. \( L_0 = 1 \)) so only one dummy state and one dummy action are defined at founder level. If, later, the model is extended to include some kind of autocorrelation of growth capacity between batches, the initial estimate for \( L \) given the previous batch may be defined as a state variable.

#### 4.2. Child level

A child process is initiated when a new batch of pigs is inserted into the pen, and it is terminated when the last pig of the pen has been delivered to the slaughterhouse and the next batch of piglets is inserted. The duration of the first stage is from insertion to the first potential date of delivery. No observations are done in the Markov process during the first stage. Accordingly only one dummy state and one dummy action are defined here. From the first potential delivery until the date of termination, stage length will be one week, unless the number of remaining pigs is 0, then stage length will also be 0 if the supply of piglets is flexible. Under constant piglet supply, the last stage (with no pigs remaining) will have a length corresponding to the time interval until insertion of the new piglets.

Assuming that delivery of pigs is considered weekly during \( n_w \) consecutive weeks, the full structure of a child process may be summarized as follows:

**Child process:** Finite time horizon corresponding to the herd life of one batch.

**Stage 1:** From insertion to the first potential date of delivery.

- **State space**: Only one dummy state is defined.
- **Action space**: Only one dummy action is defined

**Stage 2:** Stage length is one week.

- **State space**: A state is defined by the combined values of the following three state variables:
  - **Estimated scaling factor**: The current estimate \( \hat{L} \) for \( L \).
  - **Estimated sample deviation**: The current estimate \( \hat{\rho} \) for \( \rho \).
  - **Estimated standard deviation**: The current estimate \( \hat{\rho} \) for \( \rho \).
Action space: Deliver all pigs with an observed live weight exceeding $\delta_i$, $\in \Delta = (\delta^1, \ldots, \delta^n)$. The lowest possible value, $\delta^1$, is assumed to be so low that, if it is chosen, it corresponds to delivering all remaining pigs of the pen.

Stages 3, 4, $n_t + 1$: Stage length is one week as long as there are pigs left in the pen. If all pigs have been delivered, stage length is zero under flexible piglet supply. Otherwise the length corresponds to the time interval until insertion of the new piglets.

State space: A state is defined by the combined values of the following four state variables:

Number of pigs remaining: We shall denote the number of remaining pigs at time $t$ as $n_t \in \{1, \ldots, n_0\}$.

Estimated scaling factor: The current estimate $\hat{L}_t$ for $L$. 

Estimated sample deviation: The current estimate $\hat{\rho}(t)$ for $\rho(t)$. 

Estimated standard deviation: The current estimate $\hat{\delta}(t)$ for $\delta(t)$.

Furthermore, a special state, representing a situation where all pigs have been delivered ($n_t = 0$), is defined.

Action space: Deliver all pigs with an observed live weight exceeding $\delta_i$ (as for Stage 2).

Because the model is constructed within a Markov decision programming framework, the values of the state variables are split up into discrete intervals. The number of levels (i.e., intervals) are chosen by the user, and the goal is to have a sufficiently fine grained representation of values while still keeping the total number of states at a reasonable level. For a given number of levels chosen for $\hat{L}_t$ and $\hat{\delta}(t)$, the corresponding intervals are placed symmetrically around the mean in such a way that each interval has the same probability. In the examples presented later both variables are defined with 7 discrete levels. Also the number of levels for $\hat{\rho}(t)$ is chosen by the user, but in the presented examples the values of $\hat{\rho}(t)$ are distinguished at 9 levels linearly distributed from 5 to 13 kg.

To sum up, we have in the examples that $\hat{L}_t \in \{0.92, 0.96, 0.98, 1.00, 1.02, 1.04, 1.08\}$, $\hat{\delta}(t) \in \{-4.73, -2.39, -1.10, 0.00, 1.10, 2.39, 4.73\}$, and $\hat{\rho}(t) \in \{5.0, 7.0, \ldots, 13.0\}$. Thus, the total number of states at stage 2 is $7 \times 7 \times 9 = 441$. If, for instance, the number of pigs $n_t$ in the pen from the beginning is 20, the total number of states at Stages 3, 4, ..., $n_t + 1$ is $20 \times 7 \times 7 \times 9 + 1 = 8821$.

Even though no observations are made in the initial stage of the Markov decision process, the monitoring system will concurrently provide estimates for the live weights in the pen. Those estimates are used for regular updating of the parameter estimates, so that in the second stage, $t_2$, of the process, the observed values $\hat{L}_t, \hat{\delta}(t), \hat{\rho}(t)$ are based on (in principle) all observations from the time of insertion to the first potential delivery.

The threshold weights $\delta^1, \ldots, \delta^n$, defining the actions, are also user specified. In the examples they are defined as $\delta^1 = 0$ kg (corresponding to the delivery of all remaining pigs), $\delta^2 = 84$ kg, $\delta^3 = 86$ kg, ..., $\delta^{17} = 116$ kg.

5. Parameters

The rewards $r^i_i(t)$ of a stage are calculated as the economic net returns during the stage $t$ given the state $i$ and the decision $d$. In principle, the variable $i$ is the number of weeks since insertion, but since every value of $t$ corresponds to one (and only one) stage according to the model structure, we shall in the following let $t$ express the stage number.

The transition probabilities are calculated from the underlying model for live weight described in Section 3.1. At any stage, we have estimates, $\hat{L}_t + \hat{\delta}(t)$ and $\hat{\rho}(t)$ for the current sample mean and standard deviation, respectively. For a given selected threshold weight $\delta_i$ we may calculate the probability of delivering $1, \ldots, n$ pigs for slaughter as well as the conditional probability distributions for the estimates, $\hat{L}_{t+1}, \hat{\delta}(t+1), \hat{\rho}(t+1)$, at next stage.

A selection bias will occur as soon as some of the pigs have been delivered from the pen. This means that the observed sample mean $\hat{w}(t)$ will be biased. We will have to adjust for this bias under the assumption that the heaviest pigs are always delivered first and that the weight rank of a pig in the pen does not change during the relative few weeks where pigs are delivered. When there are only very few pigs left in the pen also the sample quantile $\hat{w}_{0.16}(t)$ will be biased.

Details concerning the parameters are given in the following subsections. For clarity, we shall denote the state variables belonging to state $i$ as $n_i, \hat{L}_t, \hat{\delta}(t), \hat{\rho}(t)$, respectively.

5.1. Stage lengths

The first stage, containing the dummy state, has the length, $l^0(i)$, of the number of weeks that corresponds to the time which is estimated to elapse from insertion to the first potential date of delivery. The number of weeks therefore depends on the weight of the weaners when they are inserted and is one of the setup parameters of the model (refer to Table 1). For stage 2 the stage length is always one week, i.e. $l^1(2) = 1$.

The remaining stages ($t > 2$) all have the length of one week unless the number of pigs remaining $n_0$ is zero:

$$l^i(t) = \begin{cases} 1, & n_t > 0 \\ k_t, & \text{otherwise} \end{cases}$$

where $k_t$ is under flexible piglet supply. Under constant piglet supply, $k_t$ is equal to the number of weeks until insertion of the new piglets.

5.2. Rewards

The rewards $r^i_i(t)$ of a stage are calculated as the economic gross margin during the stage $t$ given the state $i$ and the decision $d$. 

5
Basically, a reward is calculated as the expected income obtained from the pigs sent to slaughter minus the expected feeding costs and, for the first stage, the price of the piglets inserted into the pen. The expected values are conditioned on the values of the state variables and the decision made.

5.2.1. Revenues from slaughtered pigs

In week \( t \), we observe \( \hat{w}_1(t), \ldots, \hat{w}_n(t) \). We rearrange the observations in ascending order of magnitude and denote the resulting order statistics as \( \hat{w}_{(1)}(t), \hat{w}_{(2)}(t), \) and so on until the largest value \( \hat{w}_{(n)}(t) \).

When \( n_{it} < n_0 \), it means that \( n_0 - n_{it} \) pigs have already been slaughtered in previous weeks. Assuming that the heaviest pigs are always delivered first, we can regard the observed weights of the remaining pigs as observations from a truncated normal distribution (i.e. only the first \( n_0 \) order statistics \( \hat{w}_{(1)}(t), \ldots, \hat{w}_{(n)}(t) \) are available).

The decision to slaughter a pig, \( j \), will necessarily be based on the observed live weight \( \hat{w}_j(t) \). Under the action \( \delta_i \), the decision is to slaughter all pigs having an observed live weight exceeding the threshold \( \delta \). Having sorted the observations from the \( n_0 \) pigs, we logically conclude that if, for \( 1 \leq m \leq n_{it} \),

\[
\hat{w}_{(m)}(t) < \delta \leq \hat{w}_{(m+1)}(t)
\]  

(23)

it means that \( n_0 - m \) pigs are sent to slaughter with a total revenue of

\[
\sum_{j=m+1}^{n_0} w_{s,j}p(w_{s,j}),
\]  

(24)

where \( p(w_{s,j}) \) is the price per kg slaughter weight of a pig having the slaughter weight of \( w_{s,j} \).

In the optimization model we need the conditional expectation of Eq. (24) given that \( n_{it} - m \) pigs are slaughtered, i.e.

\[
E \left[ \sum_{j=m+1}^{n_0} w_{s,j}p(w_{s,j}) \mid \hat{w}_{(m)}(t) < \delta \leq \hat{w}_{(m+1)}(t) \right],
\]  

(25)

and, finally, the total expected slaughter value is found as

\[
E \left[ \sum_{m=0}^{n_{it}} \left( \sum_{j=m+1}^{n_0} w_{s,j}p(w_{s,j}) \right) \mid \hat{w}_{(m)}(t) < \delta \leq \hat{w}_{(m+1)}(t) \right] P(m),
\]  

(26)

where \( P(m) = P(\hat{w}_{(m)}(t) < \delta \leq \hat{w}_{(m+1)}(t)) \) is the probability that exactly \( m \) pigs are kept.

Even though we know that \( w_{s,j} \sim N(b\hat{w}_j, c^2\sigma^2 + \sigma^2_j) \) according to Eqs. (1) and (19), it is not trivial to evaluate the expression (25). One reason is that the price is not constant, but instead is a function of the slaughter weight. Another reason is the complicated conditioning on the order statistics of the observed live weights. The tool chosen for calculation of the expression was, in this case, a Bayesian network. For an introduction to Bayesian networks, reference is made to Jensen (2001). The key property of such a network is that inference on an unobservable hypothesis variable (in this case the slaughter revenues of the pigs) can be drawn from the values of one or more directly observable information variables (in this case the observed live weights).

A Bayesian network consists of a set of nodes, each representing a variable, and a set of directed edges connecting the variables. Each directed edge corresponds to a conditional distribution of the target variable given the origins. Methods exist for calculation of complex conditional distributions across the network.

The generic structure of the Bayesian network used in this case is shown in Figure 1 (for simplicity only three pigs are shown). Each of the three panels corresponds to a pig with the variables “olw:j” (representing the observed live weight, \( \hat{w}_{olw,j}(t) \)), “sw:j” (representing the true live weight, \( w_{s,j}(t) \)), “sw:j” (representing the slaughtered weight, \( w_{s,j}(t) \)), “Alive?:j” (having three states “Alive, kept”, “Alive, sent to slaughter” and “Already slaughtered”) and “Sent?:j” (having two states “Yes” and “No”). The utility nodes are: “Sw:j” - the resulting slaughter value of the pig (i.e. the product \( w_{s,j}p(w_{s,j}) \)), and “FC:j” - the feed cost. The two nodes labeled as “<=” connecting the three pig panels are logical constraints ensuring that the observed live weights are arranged in ascending order.

![Figure 1: The generic structure of the Bayesian network used to calculate the expected slaughter revenue and other elements of the rewards.](image)
achieved. In the examples of this paper the continuous variables are represented at 50 discrete levels each. The Bayesian network was implemented in Java using the API of the Esthauge Limid Software System. Due to the common Java environment the Bayesian network could be fully embedded in the optimization model created by the MLHMP software system (Kristensen, 2003).

All the elements of Eq. (25) can be calculated by the Bayesian network by entering appropriate evidence, propagating and extracting the relevant marginal distributions. For details about the algorithms involved, reference is made to Jensen (2001). The probability $P(m)$ of exactly $m$ pigs being kept is found by, for each pig $j$ still present in the pen, to extract $P(\hat{\omega}_j) \geq \delta^k$ which directly informs us about the probability that at most $j-1$ pigs are kept (because the values are arranged in ascending order). The probability that exactly $m$ pigs are kept is therefore

$$P(m) = P(\hat{\omega}_{(m+1)} \geq \delta^k) - P(\hat{\omega}_m \geq \delta^k).$$

(27)

Also the expression (24) which is conditioned on exactly $m$ pigs being kept, can be calculated through the following steps:

1. Enter evidence that $\hat{\omega}_{(m+1)} \geq \delta^k$.
2. Enter evidence that $\hat{\omega}_m < \delta^k$.
3. Enter evidence that the $n_i - m$ largest pigs are delivered and the smallest $m$ are kept.
4. Propagate and extract the expected slaughter value.

5.2.2. Feed costs

Also the feed costs are calculated though the Bayesian network illustrated for three pigs in Figure 1. The procedure is similar to the one used for calculation of expected slaughter revenue except that the expected feed costs are extracted. This procedure gives us the expected feed costs given that exactly $m$ pigs are kept. The overall expected feed costs are then weighed with the probabilities of keeping $m = 0, \ldots, n_i$ pigs already known form Eq. (27).

5.2.3. Price of piglets

At the first stage, the price of the piglets must be subtracted in order to calculate the reward of the stage. The price of the piglet is directly specified as input to the model.

5.2.4. Calculation of the reward

We are now ready for a full specification of the rewards, since all elements have been specified. It is simply the expected revenues from pigs sent to slaughter minus feed costs, and (for the first stage) the price of the piglets.

For the first stage the reward is further reduced by the price of the piglets inserted.

5.3. Output and other quantities

The output, $m^i(t)$, of the model is defined as the total expected live weight of pigs delivered calculated by use of the Bayesian network.

Other physical quantities may be defined as convenient for simulation purposes.

5.4. Transition probabilities

The transition probabilities $p^e_i(t)$ define the probability of transition from state $i = (n_i, \hat{L}_i, \hat{e}_i, \hat{\rho}_i)$ at stage $t$, to state $j = (n_{j+1}, \hat{L}_{j+1}, \hat{e}_j, \hat{\rho}_j)$ at stage $t+1$.

A special case is $n_i = 0$ where all pigs have been delivered. Under a flexible weaner supply, the child process will terminate with probability 1, and a new batch of piglets is inserted next week. Under a constant piglet supply, the probability of transition to the state $j$ where $n_{i+1} = 0$ will be 1.

For other values of $n_i$, the probability for a specific transition is given by the product of the probability of a change from $n_i$ pigs to $n_{i+1}$ pigs and the probability of a change from the values $\hat{L}_i, \hat{e}_i, \hat{\rho}_i$ to the values $\hat{L}_{j+1}, \hat{e}_j, \hat{\rho}_j$:

$$p^e_{ij}(t) = P(n_{i+1} | i, d) P(\hat{L}_{j+1}, \hat{e}_j, \hat{\rho}_j | n_i, \hat{L}_i, \hat{e}_i, \hat{\rho}_i).$$

(28)

The first factor in the product of Eq. (28) is already known from Eq. (27) since it corresponds to the probability of keeping exactly $n_{i+1}$ pigs. The second product is calculated from the three dimensional conditional distribution specified in Eq. (18).

5.5. Setup parameters

In order to initialize the optimization model and calculate the parameters, a number of setup parameters are needed. Those parameters are summarized in Table 1 together with the values for the examples of Section 7. In practice those values may be changed to reflect the conditions of a real herd.

The values of the first group of variables (i.e. “System information, herd and pen”) are all easy to obtain for a real herd. Most of them are facts describing the production system and/or the management strategy of the herd, even though the standard deviation of the measurement error is a property of the on-line weight assessment system.

The prior distribution of the state vector is specified through the mean vector and the variance-covariance matrix. The mean values of $L = \theta_{01}$ and $e(0) = \theta_{02}$ are 1 and 0, respectively, reflecting the natural assumption, that if nothing has been observed, the expected values correspond to the herd average expressed by $\bar{y}(0)$ in accordance with Eq. (2). The third element of the mean vector reflects the mean value of $\rho(0)$, the standard deviation inside a pen at insertion. The value 6 kg is based on experience from typical Danish herds. As concerns the variance-covariance matrix, $C_0$, it is for convenience assumed, that the elements of the prior state vector are mutually independent making all non-diagonal elements of $C_0$ zero. The diagonal elements should be estimated from herd data. The values of Table 1 only serve as examples even though they are assumed to be realistic.

2http://www.esthauge.dk
Table 1: Survey of setup parameters needed for initialization of the optimization model. The values shown are those used for the examples shown in this paper.

<table>
<thead>
<tr>
<th>Setup parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>System information, herd and pen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of pigs inserted</td>
<td>$n_0$</td>
<td>20</td>
</tr>
<tr>
<td>Average age at insertion, weeks</td>
<td>$t_0$</td>
<td>15</td>
</tr>
<tr>
<td>Weight (average) at insertion, kg</td>
<td>$w_0$</td>
<td>30.0</td>
</tr>
<tr>
<td>Std. dev. of measurement error (one pig), kg</td>
<td>$\sigma$</td>
<td>1.00</td>
</tr>
<tr>
<td>Average weekly gain - also parameter of $\gamma(t)$, kg</td>
<td>$\bar{g}$</td>
<td>6.00</td>
</tr>
<tr>
<td>Time from insertion to first decision, weeks</td>
<td>$\gamma$</td>
<td>8</td>
</tr>
<tr>
<td>Time from decision to slaughter, days</td>
<td>$\gamma$</td>
<td>3</td>
</tr>
</tbody>
</table>

Prior distribution of state vector, $\theta_0 \sim \mathcal{N}(\hat{\theta}_0, C_0)$, where $C_0 = \left[ c_{ij} \right]$

|                    |        |        |
| Scaling factor for growth capacity       | $\theta_{01}$ | 1.00 |
| Time series of sample uncertainties, kg  | $\theta_{02}$ | 0.00 |
| Standard deviation inside a pen, kg      | $\theta_{03}$ | 6.00 |
| Std. dev. of scaling factor for growth capacity | $\sigma_L = \sqrt{c_{11}}$ | 0.05 |
| Std. dev. of time series of sample uncertainties, kg | $\sigma_e = \sqrt{c_{22}}$ | 3    |
| Std. dev. of standard deviation inside a pen, kg | $\sqrt{c_{33}}$ | 1.0  |

Distribution of observation error, $z_t \sim \mathcal{N}(0, Z)$, where $Z = \left[ z_{ij} \right]$

|                    |        |        |
| Std. dev. of measurement error (average weight), kg | $\sqrt{c_{11}}$ | $\sqrt{c_{22}}$ |
| Std. dev. of observation error for quantile, kg$^3$ | $\sigma_e$ | 1.89 |
| Covariance between observation errors$^2$ | $c_{12}, c_{21}$ | 0.0024 |

Auto-regression parameters

|                    |        |        |
| Auto-regression coefficient | $\alpha$ | 0.9   |
| Standard deviation of independent random term, kg | $\sigma_e$ | $\sigma_e \sqrt{1 - \alpha^2}$ |

Live weight to slaughter weight conversion parameters

|                    |        |        |
| Conversion factor$^3$ | $c$     | 0.763  |
| Standard deviation of conversion, kg$^3$ | $\sigma_c$ | 1.4   |

Feed intake parameters

|                    |        |        |
| Energy per kg gain, FEs/kg$^4$ | $k_1$ | 1.549  |
| Energy per kg metabolic weight, FEs/kg$^4$ | $k_2$ | 0.044  |

1 From 30 kg to slaughter.
2 Determined by Monte Carlo simulation.
The variance-covariance matrix $Z$ of the observation error is partly a property of the on-line weight assessment system through the weighing precision and partly a property of samples from normal distributions. It is thus completely determined by other parameters of the model. It may be possible to establish an analytical expression for $Z$, but it is very easily determined through Monte Carlo simulation, which is used in this model. When the other parameters have been specified, a Monte Carlo simulation procedure is called automatically for determination of $Z$. The simulation procedure also determines $k$ and $a$ of Eq. (6).

The auto-regression coefficient should be estimated from herd data. The value of Table 1 only serves as a realistic example. The parameters for live weight to slaughter weight conversion are based on the officially applied value concerning the conversion factor, and a loose estimate for the standard deviation (inspired by the value used by Jørgensen (1993)). Finally, the feed intake parameters were estimated by Jørgensen (2003) by use of data from commercial Danish pig herds.

6. Integration of data flow

The design of the decision support system is illustrated in Figure 2 where also the interaction with the automatic weighing system is shown. The elements of the decision support systems are described as follows:

1. Database tables with information on:
   - The pens of the herd.
   - Information at herd level concerning usual growth rates and other biological parameters, production system (age and live weight at insertion), prices etc.
   - Pre-calculated optimal delivery policies under all possible circumstances (e.g. pen sizes and prices) for the herd and the pens. These optimal policies are calculated by use of the optimization model and the information at pen and herd level. The model must be run for each set of circumstances.
   - On-line collected weight estimates from the pens of the herd.

2. Models consisting of the following two components:
   - A dynamic linear model used for regular calculation of the current state of each pen expressed by the state variables of the optimization model.
   - The optimization model which is a two-level hierarchical Markov process with Bayesian updating as described in previous sections. This is the core of the system.

3. Intermediate and final results:
   - The current state of a pen.
   - The optimal action for a pen expressed as a threshold weight.

![Figure 2: Integration of the optimization model into the data flow in the herd.](image)

- The number of pigs to deliver from a given pen. This is the final result of the system, and it is implicitly assumed that the heaviest pigs are delivered first. This somewhat vague advice illustrates the fact that the pigs are not individually identified. It is only known how many pigs the farmer should deliver.

The data flow in the system is illustrated by arrows in the figure. A thin arrow illustrates information which is only used at setup of the system (until the conditions - e.g. the prices - change). Those arrows show the information flow used to generate the optimal solutions for the database. Many different policies must be generated because the optimal policies will depend on the number of pigs in the pen.

A bold arrow illustrates continuous data flow. The automatic weighing system produces weight estimates for a pen more or less continuously. The estimates are stored in a database and based on the registrations, the updated state of the pen may be calculated by use of the dynamic linear model. Having identified the state, the system may look the optimal action up in the database in a table of optimal policies. The optimal action is the decision to deliver all pigs (if any) exceeding a given live weight threshold. By consulting the database (the table for continuously collected weight data) the system can look up how many pigs the farmer should deliver to the slaughterhouse. Since the pigs are not individually identified, it is up to the farmer to actually identify the heaviest pigs in the pen. If, in future production systems, pigs are equipped with an identification tag, the system will even be able to tell exactly which pigs from the pen to deliver.

7. Example

7.1. Optimal policies under different scenarios

In order to illustrate the use of the model, it was created with the basic parameter values shown in Table 1 and the prices defined in Table 2. An optimal marketing policy maximizing the average gross margin per pen per week was found under different scenarios by the multi-level hierarchical Markov process,
and the technical and economical consequences of the policy were identified by Markov chain simulation. For a description of the built-in facilities of the MLHMP software system including alternative criteria of optimality and the Markov chain simulation feature, reference is made to Kristensen (2003).

The scenarios tested and the corresponding results are shown in Table 3. Figures 3 and 4 further illustrate the consequences for the average live weight at slaughter and distribution of slaughters over the weeks, respectively. As expected, the setup with flexible piglet setup leads to the highest weekly gross margin. The obvious explanation is that under those scenarios, a new batch of piglets may be inserted as soon as the pen is empty. Thus, it may sometimes be beneficial to slaughter a few remaining pigs before they have reached the (otherwise) optimal weight in order to start a new batch earlier. Under the constant piglet supply, on the other hand, slaughtering the remaining pigs has no other consequences than the pen being empty until the new piglets are inserted at the predetermined time.

Under constant piglet supply, extending the slaughter period from 3 to 9 weeks leads to a marked increase in the average live weight at slaughter, as it is seen in Figure 3. A longer production cycle means that pigs are kept until they reach the most profitable weight from a single-pig point of view. Those who have not reached the most profitable weight at the end of the slaughter period are sent to the slaughterhouse at the end of the slaughter period in order to make room for the new piglets arriving at the predetermined time. This is clearly seen in Figure 4 where the distributions of slaughters over the slaughter period are shown for the 14 scenarios defined in Table 3. The 7 curves representing constant piglet supply are more or less identical until the last delivery under each of the 7 scenarios. This reflects that the model identifies the most profitable live weight from a single-pig point of view and sends the pigs to slaughter according to that. At the final delivery any remaining pigs are slaughtered. As it is seen from the figure, more than half part of the pigs are slaughtered at the final delivery with production cycles shorter than 14 weeks. From Figure 3 it is seen, that under constant piglet supply, the most profitable length of the slaughter period is 7 weeks having as a consequence (Figure 4) that on average around 25% of the pigs are delivered in week 13.

Extending the number of weeks where slaughtering is possible from 3 to 9 has different consequences under the scenarios where piglet supply is flexible. Even though it also here leads to higher average weight at slaughter, the increase is lower and seems to converge to a value that is around 2 kg lower than under constant supply. This reflects that it occasionally happens that pigs are sent to slaughter at a lower weight in order to become able to start a new batch of piglets a week earlier.

The effect of the flexible supply is also clearly reflected in the distributions of slaughters over the slaughter period as shown in Figure 4. The shortest slaughter periods (Scenarios F-3 to F-5) are, however, so restrictive that the distributions hardly differ from the corresponding curve under constant piglet supply. For all other flexible scenarios the distribution of slaughtering is markedly different from the constant scenarios with more pigs being slaughtered early.

Extending the slaughter period means that the decision space is extended (more options are available), and accordingly, the gross margin cannot possibly decrease. This is confirmed by the “flexible” gross margin curve of Figure 3 which seems to converge to a value around 90 DKK per week. The increased gross margin even from an extension from 8 to 9 deliveries reflects that in rare cases, the pigs are growing so slowly that it is better to postpone the insertion of a new batch. In the model, such situations will occur for batches having a low value of the parameter $L$ of Eq. (2).

7.2. The learning capability of the DLM

In order to optimally utilize the online weight assessments the learning capability of the model has been given high priority through the Dynamic Linear Model used for representation of the growth process. What is learned in a DLM is the value of
### Table 2: Prices (DKK) used in the examples.

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit</th>
<th>Price, DKK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed</td>
<td>FEsv</td>
<td>1.93</td>
</tr>
<tr>
<td>Piglet</td>
<td></td>
<td>375.00</td>
</tr>
<tr>
<td>Base meat price (70.0-85.9 kg)</td>
<td>Kg slaughter weight</td>
<td>10.30</td>
</tr>
<tr>
<td>Price reduction per kg below 70 kg(^1)</td>
<td>Kg slaughter weight</td>
<td>0.10</td>
</tr>
<tr>
<td>Price reduction per kg below 60 kg(^1)</td>
<td>Kg slaughter weight</td>
<td>0.20</td>
</tr>
<tr>
<td>Price reduction per kg above 86 kg(^1)</td>
<td>Kg slaughter weight</td>
<td>0.10</td>
</tr>
<tr>
<td>Meat price from 95 to 100 kg</td>
<td>Kg slaughter weight</td>
<td>9.30</td>
</tr>
<tr>
<td>Meat price, heavy pigs (&gt; 100 kg)</td>
<td>Kg slaughter weight</td>
<td>9.10</td>
</tr>
<tr>
<td>Price adjustment, lean meat %(^2)</td>
<td>Percent</td>
<td>0.10</td>
</tr>
</tbody>
</table>

\(^1\) For each kg below/above the specified threshold the price is reduced by the value indicated.

\(^2\) For each percent above/below 61 the price is adjusted by the value indicated.

### Table 3: Scenarios and calculated key figures under optimal policies (maximizing gross margins per week). A scenario is defined by, flexible versus constant piglet supply, and maximum number of weeks where pigs may be slaughtered. In all scenarios, the first possible week of slaughter is week 9 (the pigs have been inserted at the beginning of week 1).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Gross margin, DKK</th>
<th>Av. weight at slaughter</th>
<th>Feed consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Per week Per pig</td>
<td></td>
<td>FEsv</td>
</tr>
<tr>
<td>F-3 Flex. 3</td>
<td>16.14 8.88</td>
<td>91.48</td>
<td>161.27</td>
</tr>
<tr>
<td>F-4 Flex. 4</td>
<td>55.76 33.45</td>
<td>96.63</td>
<td>176.23</td>
</tr>
<tr>
<td>F-5 Flex. 5</td>
<td>75.82 49.28</td>
<td>100.91</td>
<td>188.64</td>
</tr>
<tr>
<td>F-6 Flex. 6</td>
<td>84.65 58.91</td>
<td>103.92</td>
<td>197.13</td>
</tr>
<tr>
<td>F-7 Flex. 7</td>
<td>87.95 64.00</td>
<td>105.49</td>
<td>201.33</td>
</tr>
<tr>
<td>F-8 Flex. 8</td>
<td>89.04 66.29</td>
<td>106.14</td>
<td>202.93</td>
</tr>
<tr>
<td>F-9 Flex. 9</td>
<td>89.36 67.11</td>
<td>106.35</td>
<td>203.43</td>
</tr>
<tr>
<td>C-3 Const. 3</td>
<td>16.14 8.88</td>
<td>91.48</td>
<td>161.27</td>
</tr>
<tr>
<td>C-4 Const. 4</td>
<td>55.76 33.45</td>
<td>96.63</td>
<td>176.23</td>
</tr>
<tr>
<td>C-5 Const. 5</td>
<td>75.82 49.28</td>
<td>100.91</td>
<td>188.65</td>
</tr>
<tr>
<td>C-6 Const. 6</td>
<td>84.50 59.15</td>
<td>104.07</td>
<td>197.56</td>
</tr>
<tr>
<td>C-7 Const. 7</td>
<td>86.87 65.15</td>
<td>106.17</td>
<td>203.22</td>
</tr>
<tr>
<td>C-8 Const. 8</td>
<td>85.81 68.65</td>
<td>107.49</td>
<td>206.55</td>
</tr>
<tr>
<td>C-9 Const. 9</td>
<td>82.94 70.49</td>
<td>108.23</td>
<td>208.33</td>
</tr>
</tbody>
</table>
the unobservable parameter vector $\theta$, which in this case has the elements $L$ (the relative growth capacity), $e(t)$ (the temporary deviation) and $\rho(t)$ (the within-pen standard deviation). Thus, for instance, a fast growing and homogenous batch will have a high value of $L$ and a low value of $\rho(t)$. From the online weight estimates, the model will - over the lifetime of the batch - adapt to the unobservable, true values.

In order to illustrate the learning capability, a Monte Carlo simulation procedure was added to the software plug-in. The simulation procedure is able to randomly generate observed time series corresponding to observations from the weighing equipment from insertion to slaughter under predefined true values of the relative growth capacity $L$ and the within-pen initial standard deviation $\rho(1)$. Having simulated a time series, we can week by week update the current estimates $\hat{L}_t$ for $L$, and $\hat{\rho}(t)$ for $\rho(t)$. The advantage of using simulation instead of real data for this purpose is that the true values for $L$ and $\rho(t)$ are actually known. In Figure 5 two examples of the stepwise learning are shown.

The first example (Figure 5(a)) shows how an underlying true value of the relative growth capacity ($L = 1.04$) is learned from data. As it is seen, the estimated value initially is close to the population mean (i.e. the value 1), but moves fast in the direction of the true value for the pen. After some weeks, the adaptation appears more arbitrary, probably because the distinction between the temporary autocorrelated sample error $e(t)$ and $L$ is difficult, but nevertheless, the estimate is close to the true value at the end of the period.

In Figure 5(b), a corresponding example of learning the homogeneity of the pigs in the pen is shown. An unusual high true value is assumed (the linear growth is due to Eq. (7)), and the estimated value is week by week approaching the true value.

Even though Figure 5 illustrates learning in a very direct way, the figure suffers from the weakness that it only shows single examples of simulated time series corresponding to weighing results from single batches. An indirect, but more systematic approach, is to simulate multiple replications of time series under the same conditions (true values of $L$ and $\rho(t)$) and compare the actual consequences for the delivery policies under different conditions.

The simulation was carried out with 10000 replications for each combination of piglet supply (scenario F-9 versus C-7), relative growth capacity (7 levels: $L = 0.92, 0.96, 0.98, 1.00, 1.02, 1.04, 1.08$) and homogeneity (within-pen standard deviation $\rho(1) = 3, 6, 9$). The consequences of the true characteristics of the pigs are illustrated by the average distribution of deliveries over weeks.

In Figure 6 the effects of different relative growth capacities, $L$, are shown under flexible and constant piglet supply. It must be emphasized that the optimization model (of course) does not know the true value of $L$ (since it is unobservable in practice). It is only because we use simulated data that the value is known in this case. The optimization model solely relies on the weekly updated estimates, $\hat{L}_t$, $\hat{e}(t)$ and $\hat{\rho}(t)$.

The very different delivery profiles in Figure 6(a) illustrate that the learning algorithms of the DLM works very well in the sense that the optimization model is really able to identify the
7.3. Economic value of increased weighing precision

Since the model has has been developed independently of the automatic weighing system an arbitrary weighing precision has been used in the previous examples. The precision will
obviously vary from system to system so it will be relevant to investigate the influence of the weighing precision on the gross margin thus assessing the value of precision. For developers of such weighing systems this information is relevant for the decision of what level of precision to aim at.

In order to calculate the economic value of increased weighing precision, the standard deviation of the observation error ($\sigma$ in Table 1) was varied in steps of 0.5 between 0.0 and 3.0. The value $\sigma = 0.0$ corresponds to the unrealistic assumption of no observation error at all. It serves as a theoretical upper limit for the value of precision. The scenarios F-9 and C-7 were used for these tests.

The results of the sensitivity analysis showed that the weighing precision has only a very modest influence on the economic result. Thus, a reduction of the standard deviation of the measurement error from 3.0 kg to 0 kg only improved the net returns per pen per week from 89.03 DKK to 89.41 DKK under flexible piglet supply and from 86.54 DKK to 86.95 DKK under constant piglet supply.

8. Discussion

The novel contribution of the present model to the solution of the optimal slaughter pig delivery problem is first of all that it utilizes online weight assessments of the pigs throughout the fattening period. The weight assessments are used to concurrently update the estimated growth potential, $L_{t}$, of the present pigs, their temporary deviation from the growth curve, $\hat{e}(t)$, and their homogeneity, $\hat{\rho}(t)$, in order to have the best possible basis for prediction of the future growth of the pigs. The weight assessments may also be used to inform the farmer about how many pigs to deliver from a given pen at the next delivery.

As concerns the performance of the model, it seems to work appropriately. The Monte Carlo simulations confirm that the learning capability of the model enables it to adapt to true values of growth capacity and homogeneity in a fast and efficient manner. The Markov chain simulations, furthermore, confirm that the optimal policies seem to reflect the setup conditions concerning piglet supply and number of weeks of delivery in a natural way.

No previous studies known to the authors have included information from online weighing systems at pen level.

The study by Niemi (2006) had a different focus with emphasis on potential benefit of technological changes in production and the system being modeled was one pig (i.e. not a group) assuming known body composition. Thus, it hardly makes sense to compare his model to the one presented here.

Jørgensen (1993) also modeled one pig implying that the aspects concerning the decision to slaughter all remaining pigs in a pen are not handled. The focus of the study was to estimate the value of weighing precision, and automatic equipment was also considered even though no learning algorithm was included in the model. Even though the study was carried out on single-pig level the conclusion of the study was the same as in Section 7.3 that the economic value of increased precision is rather low.

Studies by Kure (1997), Roemen and de Klein (2000), Toft et al. (2005), Boys et al. (2007) and Ohlmann and Jones (2008) resemble the present in the sense that groups of pigs were modeled instead of just one pig. Thus, both dimensions of the problem (delivery of individual pigs and final delivery of all remaining pigs) were treated. Kure (1997) as the only one even included a learning aspect, but only based on historical data from the herd. No online learning from automatic weighing equipment was included in the study. Ohlmann and Jones (2008) also included decisions about to which packer to sell the pigs, but under Danish conditions with farmer owned cooperative slaughterhouses such an optimization has no relevance because the farmer is legally bounded to deliver to the cooperative.

Surprisingly the optimization results showed that under the conditions imposed by the setup parameters of Table 1, it is optimal to deliver pigs for slaughter over at least 7 weeks. There is no doubt that it would be difficult to find a farmer following policies like those identified as optimal in this study. Several reasons can explain the mismatch between theory and practice. Farmers may underestimate the economic benefit of fine tuning the policy or the within-pen standard deviation used in this study may be overestimated. A third explanation could be that this study assumes that there are no costs associated with sorting out pigs and sending them to slaughter. This may not be true in practice. If the $k$ heaviest pigs are to be sorted out and sent to the slaughterhouse some kind of labor input is required. In the long run such an effort has a cost and should be accounted for. Still, however, the number of deliveries to slaughter should be part of the optimization. A solution could be to include in the model a constant cost of delivery, independently of the number of pigs sent to slaughter. This would consequently make many deliveries less profitable. Since the probability of sending $k > 0$ piglets to slaughter is known from Eq. (27) such an extension to the model could be handled.

The presented model is intended as the engine of an integrated decision support system for a slaughter pig herd. It is not known yet how the logistics of such a monitoring system will be implemented. In the previous sections it has been assumed that for the pen considered, we have weight estimates. It is probably not realistic to expect that a monitoring system is available in all pens. The most likely solution is that only a few representative pens are monitored so that we have to draw inference on the remaining pens given the observations from those being monitored. More consideration is needed in order to implement a model for a pen not being monitored.

As the model has been formulated in this study, it is assumed, that the estimates $L_{t}, \hat{e}(t), \hat{\rho}(t)$ are only updated once per stage (i.e. once a week). Since, however, the monitoring system produces observations more or less continuously, it would be possible to update the estimates more often - for instance daily. Because the number of pigs is constant between two deliveries, it would be possible to extend the model to a second child level representing the time period between two deliveries. The stage length at this new level would be one day, and the state variables would be the three estimates $L_{t}, \hat{e}(t), \hat{\rho}(t)$. The decision concerning the selection of the threshold weight $\delta_{t}$ would have to be moved to this level at the stage where the delivery decision is made (a few days before the actual delivery, because the slaughterhouse must be notified in advance). There would, in
principle, be a separate process for each state at child level 1, but since only the initial distribution of $L_0, \hat{c}(t), \hat{p}(t)$ differs between states having the same number of pigs left, we only need a process for each value of $n_t$ by use of the “sharing action” facility of the MLHMP software (Kristensen, 2003).

For simplicity, the current model does not take the risk of death into account. In practise around 4% of the slaughter pigs die during the finishing period (Vinther, 2011). Extending the model with a weekly death risk corresponding to the death rate of the entire period would be quite simple.

The model assumes that the growth of the pigs that remain in the pen is initially independent of the number of pigs still in the pen. On the other hand, if they actually grow faster when there are fewer left in the pen, the model will adapt to the improved growth rate due to the learning capability of the DLM. Still, however, it is not expected ex ante. Currently we do not have reliable data to determine the relationship between stocking density and growth rate, but in the future such knowledge will probably become available at herd level as automatic weighing systems become common. It would then be possible to take it more actively into account in a model like this one.

Recent research (Jørgensen et al., 2011) describes how to integrate graphical models like decision graphs into hierarchical Markov decision processes. It should also be considered whether such a combined technique could be of use for a problem like the one presented in this paper.

9. Conclusions

The developed optimization model is able to adapt to the growth and homogeneity properties of the individual batch presently occupying the pen. Furthermore, the model reflects the setup conditions in a natural way leading to lower live weight at slaughter and higher net revenue per week if the supply of piglets is flexible as opposed to constant (with piglets being inserted with predefined intervals). The optimal policies result in delivery of pigs over more weeks than usually seen in practice. This is probably partly due to farmers underestimating the economic benefit of fine tuning the slaughter policy and partly due to the fact that the model in the current version ignores the labor costs of sorting pigs out for slaughter. Finally the economic value of weighing precision was found to be low when the standard deviation of the observation error was varied from 0 to 3 kg. Thus, the demand for accuracy of the automatic weighing system can be relaxed accordingly.

10. Acknowledgements

The authors wish to acknowledge the Danish Ministry of Food, Agriculture and Fisheries for financial support for this study through a grant entitled Weighing by image analysis - Development of methods for weight assessment in pigs by image analysis.


URL: http://ajae.oxfordjournals.org/content/89/1/24.abstract
