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Optimal replacement policies and economic value of clinical observations in sow herds

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Abstract

Even though several sow replacement models have been published, integrating information about the health status of sows has not yet been handled satisfactorily. This paper presents a framework for integrating a Weak Sow Index (WSI) into an existing sow replacement model. The WSI, which is developed as part of this study, quantifies various clinical signs into one numerical value representing the risk of a sow to be involuntarily culled. The objective of the study is to investigate the effect of observing clinical signs of sows on the optimal replacement policy. A second objective is to estimate the economic value of observing clinical signs of individual sows. Bayesian networks are used to develop the WSI models for lactating and pregnant sows, respectively. The optimization of the replacement policy is done in a multi-level hierarchical Markov decision process. To illustrate the behaviour of the model, the effect of the WSI on the replacement policy, and the economic benefit of observing clinical signs of individual sows are investigated in two fictitious herds with a high and a low risk of involuntary culling of sows. In general, the value of the WSI has a high influence on the optimal replacement policy, allowing for a better economic classification of sows when taking information about the health status into account. It is shown that the economic value of the WSI is higher in a high risk herd compared to a low risk herd. Among the individual clinical signs, "unwillingness to stand" made the lowest contribution to the economic value of the WSI. The highest contribution was made by the clinical sign "vulva bite" in pregnant sows.

Keywords: Sows; Replacements; Clinical observations; Economic value

1. Introduction

The sow replacement problem is today one of the most important challenges in sow herd management. Sow replacement decisions influence the expected lifetime of sows, the annual replacement rate, the piglet production capacity and other important key figures for planning of the pork supply chain. The replacement decisions do not only have a direct economical impact for the farmer, but also for the pork supply chain where herd production is integrated. Replacement of sows is basically defined as a management decision determining the optimal time to replace a sow, based on the characteristics of individual sows (e.g. parity, litter size and conception rate) (Kristensen, 1994). Several quantitative models focusing on the sow replacement decision have been developed and described in literature. Generally, those models try to model the dynamic production of sows taking into account the most representative and directly observable variables such as the conception rate, the litter size and the genetic merit (Plà, 2007).

However, optimal replacement policies incorporating animal health aspects are becoming increasingly important in modern sow farming. Until now no replacement models have incorporated information about the health status of individual sows, probably due to difficulties in observing and recording variables representing the health status, such as clinical signs and physical and behavioural abnormalities, in a systematic and operational way. A poor health status (defined as the occurrence of clinical signs and/or the presence of physical or behavioural abnormalities) can cause failure to conceive and increase the risk of abortion and sow mortality (Christensen et al. 1995). Indeed a poor health status of sows also has a negative impact on the animal welfare. The incorporation of health indicators in sow replacement models seems to be crucial since a poor health status has a major impact on sow farm production, and hence, economy (Straw et al., 2006).

The aim of this study is to incorporate information about the health status of individual sows into an existing sow replacement model developed by Kristensen and Søllested (2004a, b). The health status of a sow is described as an index (called the Weak Sow Index (WSI)) quantifying significant clinical signs into one numerical value. The effect of the health status of sows in both the gestation and lactation period on the optimal replacement policies will be studied.

The outline of the paper is as follows. First the development of the WSI will be described and the integration of the WSI into the replacement model presented. To illustrate a potential application of the model, a fictitious herd with different risk levels of involuntary culling of sows will be used as an example. By varying the observation policy of clinical signs, the economic benefit of each clinical sign when deciding on replacement of a sow is calculated. Thus, it is possible to balance the value of information against the labour requirements.

2. Materials and Methods

2.1 The existing replacement model

The existing replacement model is a multi-level hierarchical Markov process using Bayesian updating (Kristensen and Søllested, 2004a, b). In general, a hierarchical model is an infinite stage Markov decision process with parameters defined in a special way, but nevertheless in accordance with all usual rules and conditions relating to such processes. The basic idea of the hierarchical structure is that stages of the process can be expanded to a so-called child process, which again may expand stages further to new child processes leading to multiple levels. By using hierarchical modelling more complex models can be solved (Kristensen and Jørgensen, 2000).

The litter size potential of the sow calculated by Bayesian updating and the number of re-matings are the properties (state variables) represented in the original model by Kristensen and Søllested (2004a, b). Furthermore, the age of the sow is represented through the hierarchical structure. The Bayesian updating technique applied in the model is based on a dynamic linear model as described by West and Harrison (1997). The transition probabilities are based on a litter size model of which the parameters are fitted as described by Toft and Jørgensen (2002).

In order to incorporate the health status of sows in the existing replacement model, we first of all develop a WSI for a sow quantifying various clinical signs into one numerical value representing the risk of a sow to be involuntarily culled.

2.2. Development of the WSI

The tool used for quantifying several clinical signs into one numerical value was a Bayesian network. For an introduction to Bayesian networks, reference is made to Jensen (2001). The key property of such a network is that inference on an unobservable hypothesis variable (in this case the WSI) can be drawn from the values of one or more directly observable information variables (in this case the clinical signs).

2.2.1. General structure of the WSI

The WSI is constructed by use of a Bayesian network consisting of a set of herd level variables, H, a set of sow specific variables, S, representing potentially observed clinical signs, a sow specific variable n representing the parity, and finally, a sow specific variable w representing the WSI. The network further has the property that, for any variable $x \in H$, x is d-separated (Jensen, 2001) from w given $S \cup \{n\}$. The practical implications of the d-separation, in this particular case, are that the WSI, w, is calculated exclusively from clinical observations at sow level, i.e. a subset of S, but since the prevalence of the clinical signs depends on herd specific conditions, the distribution of w will be herd specific. Thus, the transition probabilities describing the dynamic properties of w will be herd specific as well (those transition probabilities basically define the probabilities of having a certain value of w in a given stage of the reproductive cycle given a known value of w in the previous stage) .

All variables in the Bayesian network are discrete. The herd level variables are categorical with a number of mutually exclusive states characterizing the production system (for instance the variable “herd size” has the state space {< 400 sows, 400-600 sows, > 600 sows}). The clinical signs are typically binary or ordinal. The WSI, w, is numerical, and since it is later used as state variable in a Markov decision process, it is modelled as a number of discrete numerical levels.

Two versions of the network, one for the gestation period and one for the lactation period, are constructed. In general, the WSI represents the risk of a sow to be involuntarily culled (within 3 months after the clinical examination); however the interpretation of involuntary culling differs for pregnant and lactating sows. Initially, the intention was that "involuntary culling" should represent sows that have died unassistedly or have been euthanized. For the lactating sows, this definition was kept, but for the pregnant sow, “involuntary culling” represents a pool of sows that have died unassistedly, been euthanized or sent to slaughter unexpectedly (due to clinical signs and/or physical and behavioural problems). The reason for this conceptual extension for pregnant sows was entirely of practical nature, because the number of dead/euthanized pregnant sows was too small to establish significant relations between the clinical signs and the risk of involuntary culling. Therefore, sows sent to slaughter unexpectedly were included in the pool of involuntarily culled sows.

Sows which were culled voluntarily were not included, because the decision to cull a sow is actively determined by the optimization model.

2.2.2 Construction of the WSI for lactating and pregnant sows

A reproductive cycle of a sow consists of a mating period (usually only a few days from weaning to pregnancy), a gestation period (115 days) and a lactation period (usually around 4 weeks). Due to the short duration of the mating period, the WSI has only been defined for the gestation and the lactation period.

The models are developed using data from 3541 pregnant and 1347 lactating sows, randomly selected from 34 Danish sow herds (Jensen et al., 2010). Individual examination of each sow was performed by technicians using a protocol of 16 clinical signs (e.g. lameness {no, mild, severe}, body condition score {below average, average, above average} and vulva bite {no, yes}). After the clinical examination, farmers from each herd recorded all cullings of sows (euthanization, sudden death or sent to slaughter). Only culling information recorded maximum 3 months after the clinical examination are used for the construction of the WSI models. From an explanatory factor analysis, Jensen et al. (2010) found 3 latent factors to incorporate a number of clinical signs observed in the gestation unit that shared a common structure. These factors were interpreted as: “Pressure marks”, “Wounds” and “Lameness”. The WSI model for pregnant sows is modelled by combining the clinical signs, the 3 latent factors and the probability of involuntary culling. To identify and estimate significant links in the model, logistic regression analyses are performed with “involuntary culling” as outcome variable and the latent factors, as well as clinical variables not included in the factors, as explanatory variables. Only the factor: “Lameness” (which included the clinical signs lameness {no, mild, severe} and unwillingness to stand {no, yes}) is found to be significantly associated with the outcome variable: “Involuntary culling” ($p=0.01$). Moreover, the clinical sign: vulva bite {no, yes}, which does not load high on any of the 3 factors, is significantly associated with “Involuntary culling” ($p<0.05$), and is therefore included in the WSI model for pregnant sows (Figure 1). Thus, the set of sow variables for the pregnancy period is the set of three clinical signs S = {"lameness", "unwillingness to stand", "vulva bite"}.

Analogously, the WSI for the lactation period is developed. From the explanatory factor analysis, 2 factors extract the clinical signs of lactating sows that describe the variation in data (data not shown). These factors are interpreted as “Pressure marks” and “Wounds”. Based on a logistic regression analysis, the 2 latent factors do not affect “Involuntary culling” significantly. However, the clinical signs: Vulva colour {no, yes}, shoulder ulcer {no, scar, ulcer} and body condition score {below average, average, above average} all have a significant effect on “involuntary culling” ($P<0.05$), and are therefore included in the WSI for lactating sows (Figure 2). Thus, the set of sow variables for the lactation period is S = {"vulva colour", "shoulder ulcer", "body condition score"}.

Information about the herds (e.g. herdsize {< 400 sows, 400-600 sows, > 600 sows}, feeding system {electronic sow feeding, feeding boxes, competition based feeding} and deep bedding {yes, no} (pregnant sow)) is finally investigated for inclusion in the WSI models. Based on logistic analyses, the herd variables "herd size" and "feeding system" are found to affect the clinical variables "lameness" and "unwillingness to stand", whereas "deep bedding" is found to influence the occurrence of "vulva bite" in a sow. Hence, these herd risk factors are included in the WSI for the pregnant sows (Figure 1).

In other words, the set of herd level variables for the pregnancy period is H = {herd size, feeding system, deep bedding}.

For the lactating sows, herd size and feeding system are found to influence shoulder ulcer only, and are consequently included in the WSI model for lactating sows (Figure 2). Thus, the set of herd level variables for the lactation period is H = {herd size, feeding system}.

No effect of parity is found in any of the analyses. Nevertheless, it is decided to keep parity in both networks in order to illustrate the full potential of the developed framework. The conditional probabilities of the WSI, w, are just defined independently of the parity.

The parameters used for calculation of the conditional probabilities of the Bayesian networks are shown in Appendix A. The Bayesian networks were implemented by use of the Esthauge LIMID software system[1] using the variables and parameters of Tables A.1 and A.2. Given a set of observed clinical signs of a sow, the implemented Bayesian networks shown in Figures 1 and 2 will calculate the corresponding WSI.

2.3 Use of the Bayesian networks for calculation of the WSI.

We assume that all variables in H are observed in every herd, whereas the sow specific variables being observed depend on an *observation policy* defining the clinical examinations done in the herd. Denote as $E_h \subseteq S$ the set of sow specific clinical variables being observed in herd h. A corresponding configuration (or evidence set) of E_h observed for sow s is denoted as e_{hs}.

In principle, the WSI for a sow is found by entering the observed values of the sow variables (the clinical signs) and then propagating. To propagate simply means to find

[1] www.esthauge.dk

the conditional distribution of an unknown variable (in this case *w*) given observed values of other variables (in this case the clinical signs). For a detailed description including algorithms, reference is made to Jensen (2001). If all sow variables are observed (i.e. if $E_h = S$), the value of *w* is known with certainty. If fewer sow level variables are observed (i.e. if $E_h \subset S$), an estimate for the WSI is still available from the Bayesian network, but the precision will be lower (the economic consequences of different levels of precision are estimated later in Section 3.3). In the replacement model, the WSI is expressed relatively as the deviation from the herd average (defined by the values of the herd level variables in *H*).

Moreover, we assume the observed WSIs for the gestation and lactation period of an individual sow to be autocorrelated as a first order autoregressive time series.

Denote as w_{ns}^G the WSI for the gestation period of sow s at parity n in herd h. We may model the value as a sum of an underlying herd mean μ_{hn}^G and a random term A_{ns}^G:

$$w_{ns}^G = \mu_{hn}^G + A_{ns}^G \qquad (1)$$

Where the sow specific variable A_{ns}^G has the properties $E(A_{ns}^G) = 0$ and $V(A_{ns}^G) = \sigma_{Ghn}^2$.

The herd mean, μ_{hn}^G of Eq. (1), is found as

$$\mu_{hn}^G = \mathrm{E}(w \mid h, n) = \sum_{j=1}^{J} P(w = w_j \middle| h, n) w_j \,, \qquad (2)$$

where, now, h is interpreted as the configuration of *H* representing the observed values of the herd level variables, and w_j is the value corresponding to the j th discrete level of w. The expected value is simply found by inserting h and n as evidence in the

Bayesian network and then propagating. The variance σ^2_{Ghn} between sows in the herd is found analogously.

For a given configuration e_{hs} of the observation set, the estimated (relative) WSI, $\hat{w}^G_{ns}$, of the sow will be defined as

$$\hat{w}^G_{ns} = E(w^G_{ns} | e_{hs}, n) - \mu^G_{hn} \quad . \qquad (3)$$

The precision of this estimate depends, as mentioned, on the herd observation policy described by E_h. For convenience the standard deviation will be denoted as σ_{Gn}, where

$$\sigma^2_{Gn} = Var(w^G_{ns} | E_h, n) \quad . \qquad (4)$$

Thus σ_{Gn} may be regarded as the standard deviation of the observation error. It should be noticed, that this approach assumes variance homogeneity over the configurations of E_h. For the special case where all sow specific variables (described in section 2.1) have been observed (i.e. if $E_h = S$) there will be no observation error, implying that $\sigma_{Gn} = 0$. The conditional expectation and variance in Eqs. (3) and (4) are found by inserting the evidence into the Bayesian network followed by a propagation. Thus, assuming normal distributions,

$$(w^G_{ns} | e_{hs}, n) \sim N(\mu^G_{hn} + \hat{w}^G_{ns}, \sigma^2_{Gn}) \quad . \qquad (5)$$

Eq. (5) is used as basis for deduction of the transition probabilities of the replacement model.

The (relative) WSI for the lactation period, $\hat{w}^L_{ns}$, is modelled completely analogously.

2.5. Integration of the WSI into the replacement model.

2.5.1 State variables for the WSI

The WSI state variables will in the replacement model be represented at $2k+1$ levels, $-k,...,0,...,k,$ where $-k$ corresponds to a very weak sow, 0 to a sow at the herd average for the parity (and stage of cycle), and $+k$ corresponds to a very strong sow. Hence, in the replacement model, the WSI of a parity n sow s is represented by state variables as follows:

In the gestation period: The estimated WSI for present gestation period, $\hat{w}_{ns}^{G}$.

In the lactation period: The estimated WSI for present lactation period, $\hat{w}_{ns}^{L}$

For parity $n > 1$**:** The estimated WSI for previous lactation period, $\hat{w}_{n-1,s}^{L}$. This state variable is necessary for representation of the autocorrelation between the WSI in previous lactation period and present pregnancy period.

Combining these new state variables with those included by Kristensen and Søllested (2004a, b) results in a hierarchical model with 3 levels. In the description below, a state space is either defined by one or more state variables or it is enumerated. The two definitions are also sometimes combined. If a state space only contains one state (which accordingly has probability 1) it is referred to as a "dummy" state. Action spaces are enumerated, and if only one action is defined (which accordingly always is chosen) it is referred to as a "dummy" action. The full model is defined as follows:

Founder process: Infinite time horizon.

> **Stage:** Stage length is equal to the life span of a sow in the herd.
>
> **State space:** Only one dummy state is defined.
>
> **Action space:** Only one dummy action is defined.

Child level 1: Finite time horizon.

Stage: Stage length is equal to a reproductive cycle from weaning to weaning. Stage number equals parity.

State space: Depends on parity:

Parity 1: Only one dummy state is defined.

Parity >1: Two state variables are defined:

- Litter size potential (21 levels)
- WSI of previous lactation period (2*k*+1 levels).

An additional state representing a culled sow is added. The number of states equals 21×(2*k*+1) +1.

Action space: Mating method: 2 options that for instance represent "Normal mating" and "Artificial insemination" as described by Kristensen and Søllested (2004a, b).

Child level 2: Finite time horizon.

Stage: Stage length is equal to the duration of "Mating" (stage 1), "Gestation" (stage 2) or "Lactation" (stage 3).

State space: Depends on stage:

Stage 1, "Mating": Three states reflecting health status: "Healthy", "Diseased" and "Dead". The "Diseased" state is used to represent involuntarily culled sows which can still be slaughtered, whereas "Dead" represents dead and euthanized sows.

Stage 2, "Gestation": "Pregnant" with a combination of WSI level of the current gestation period. Two additional states representing a "Diseased" and "Infertile" sows are added. The number of states equals (2*k*+1) +2.

Stage 3, “Lactation”: Two state variables are defined:

- “Litter size”, present parity (20 levels).
- WSI of present lactation period ($2k$+1 levels).

Two additional states representing a “Diseased” and “Dead” sow are added. The number of states thus equals 20×($2k$+1) +2.

Action space: Depends on stage:

Stage 1, “Mating”: Mating policy: Allow 1,…,5 matings before culling for infertility if the sow is “Healthy”. If the sow is “Diseased” or “Dead”, only one dummy action is defined.

Stage 2, “Gestation”: Only one dummy action is defined.

Stage 3, “Lactation”: Two actions defined: “Keep the sow” and “Replace the sow at weaning”. But if the sow is “Diseased” or “Dead”, only one dummy action is defined.

The model has been implemented as a plug-in to the MLHMP software system presented by Kristensen (2003).

2.5.2 The effect of the WSI for the individual sow

A pregnant sow with a high WSI increases the probability of sudden death, euthanization or sent to slaughter due to a poor health status. Each level of the (current) WSI will be associated with a value on the logistic scale reflecting directly the probability of death/euthanization/sent to slaughter due to a poor health status. As an illustration it can be mentioned that a pregnant sow with clinical signs ("unwillingness to stand", "lameness", "vulva bite") = ("no", "no", "no") will have WSI of -3.314

corresponding directly to a probability of involuntary culling of 0.0351. In contrast, a sow with clinical signs ("unwillingness to stand", "lameness", "vulva bite") = ("yes", "severe", "yes") will have a WSI of -2.084 corresponding to a probability of involuntary culling of 0.111.

The state space at child level 2 will, as described above, have a special state called “Dead” reflecting that the sow dies suddenly/is euthanized. The value of the dead sow is zero, but there will be an additional disposal cost. The state “Dead” will be defined for the mating period and the lactation period. The state “Diseased” will in the extended model only be used for an involuntarily culled sow which can be sent to slaughter (and thus have a full slaughter value).

For lactating sows, a high WSI increases the probability of sudden death or euthanization, but not the probability of sent to slaughter due to a poor health status. Hence, each level of the (current) WSI will be associated with a value on the logistic scale reflecting directly the probability of death/euthanization. Additionally, the WSI for sows in the lactation period will influence the conception rate of the following mating period.

2.6 Description of the parameters.

2.6.1 Information needs for model construction

The basic idea behind the existing sow replacement model described by Kristensen and Søllested (2004a, b) is that a herd specific model is constructed. The probabilities of the Bayesian networks and the necessary autocorrelation coefficient for the WSI from the gestation to the lactation period ρ_{GLn} are presented in Appendix A. Due to few repeated measurements from the lactation period to the next gestation period, it is not possible to estimate the corresponding autocorrelation coefficient, ρ_{LGn}, and we therefore assume that $\rho_{LGn} = \rho_{GLn}$.

In order to adapt the model to the conditions of a specific herd, we need information about the values of all variables in *H* for the herd, and information about the observation policy both in the gestation and lactation period (i.e. identification of $E_h \subseteq S$).

2.7. Transition probabilities

2.7.1 The WSI probabilities

The transition probabilities from state *i* to state *j* express the combined probabilities of the transitions reflected in the values of the state variables belonging to state *i* and those belonging to state *j*. The final transition probabilities are calculated as the product of the individual transition probabilities for the state variables in question.

The probabilities related to transitions in litter size potential, observed litter size and conception are calculated as described in the original articles by Kristensen and Søllested (2004a, b). For the extended model, the probabilities related to transitions in the WSI are needed (Table 1). The precise calculation of those probabilities is described in details in Appendix B.

The final transition probabilities are a combination of the transition probabilities originating from the WSI and those from the litter size model and the mating policy model already described in Kristensen and Søllested (2004a, b). They are all defined at Child level 2 of the model. A process at child level 2 has 3 ordinary stages for mating, gestation and lactation, respectively, and in addition an initial dummy stage holding the probability distribution over states of the mating stage. For a gilt (parity 1) only the information defined by the states at child level 2 is available. Sows from parity 2 and higher are also characterized by the state variables defined for child level 1 (the WSI of previous lactation period and the litter size potential).

In the following description we shall denote as $p_{ij}^{d}(n)$ the probability of transition from state i at stage n to state j at stage n+1 under decision d.

2.7.2 Initial state probabilities (Stage 0)

For a gilt, no WSI information is available before mating, so the probabilities are defined as in the current model described by Kristensen and Søllested (2004a, b).

For sows in parity 2 and higher, information about WSI of the previous lactation period is available. This information is stored in the Child level 1 state. The initial transition probabilities $p_{1j}^{1}(0)$ define probabilities to stage j of the mating stage, where $j \in$ {“Healthy”, “Diseased”, “Dead”}.

For j=”Healthy”, the probability equals the probability of the sow not being involuntarily culled. The calculation of the probabilities j=”Diseased” and j=”Dead” is described in details in Appendix B.

2.7.3 Mating period (Stage 1)

Ignoring the special states, “Diseased” and “Dead” there is only one state i to consider, i.e. i=”Healthy”. The 5 actions “Allow d matings” must define transition probabilities to states representing different values of WSI. Let $d \in \{1,\ldots,5\}$ be the action of the mating period, and let $j \in \{1,\ldots,2k+3\}$ be the state of the gestation period. The value of j corresponds directly to level of WSI. If the model has more than one mating method, the actual mating method is known from the decision at Child level 1.

For a gilt, no information about previous WSI is available and the calculation is rather simple. Defining the events S and C corresponding to survival and conception, respectively, the transition probability $p_{ij}^{d}(1)$ is then, for $j < 2k + 2$, calculated as the product of the probability of conception, the probability of survival (neither dead nor diseased) and the probability of observing a certain WSI level in the gestation period. The probabilities of conception and survival are the same as described by Kristensen and Søllested (2004a, b), and the probability of WSI level is described in Appendix B. The probabilities of "Infertile" and "Diseased" are as described by Kristensen and Søllested (2004a, b).

For a parity 2 sow (and higher), information about the WSI from previous lactation periods is available, and stored in the Child level 1 state. This value influences the conception rate and the WSI of the gestation period. The transition probability $p_{ij}^{d}(1)$ at parity n is again, for $j < 2k + 2$, calculated as the product of 3 separate probabilities (conception, survival and WSI level). However, the transition probability of the WSI level and the conception rates are now conditioned on WSI level of previous lactation period. Appendix B describes the calculation of the conditional probability.

Probabilities of the two states "Infertile" and "Diseased" are calculated in the same way as for parity 1.

2.7.3 Gestation period (Stage 2)

Ignoring the special states ("Diseased" and "Dead") the other states are described directly by their WSI level. The probabilities link to states j of the lactation period, where a state is described by the present litter size and the WSI level.

The transition probabilities for the gestation period are calculated as the product of the conditional probability of observing a given litter size (given litter size potential known from child level 1), the conditional probability of survival (given current WSI level) and the conditional probability of observing a given WSI level in the lactation period (given the current WSI level). The transition probability related to litter size is as described by Kristensen and Søllested (2004a, b), whereas the two other conditional probabilities are described in Appendix B. Additionally, the calculation of the probabilities of the states “Dead” and “Diseased” is described in Appendix B .

2.7.4 Lactation period (Stage 3)

Here, the state i is described by combined values of litter size and WSI. For the action “Keep”, the states, which the probabilities link to, are the Child level 1 states of the next parity. A destination state j^1 (at Child level 1) is described by combined values of updated litter size potential m_{j^1} and old WSI, $\hat{w}_{nj^1_w}$. The updated litter size potential of next parity is just the current value updated with the present litter size, and the old WSI of next parity is simply the present WSI. Thus, the destination state $j^{1'}$ for Child level 1 at next parity is known with certainty, and the transition becomes deterministic in the sense that, for this state, $p^d_{ij^{1'}}(3)=1.0$.

For the highest parity the transition probabilities define a deterministic transition to the founder stage.

Under the action “Replace” the process goes to state “Replaced” at Child level 1 with probability 1.0.

2.8 Rewards

The rewards of the model are calculated in the same way as it was described by Kristensen and Søllested (2004b), except for the states ”Diseased” and “Dead”. If the sow is dead (or has been euthanized), the farmer does not receive any income, but will have to pay a cost for disposal of the dead body. If the sow is diseased, it is send to slaughter immediately, and the reward equals the slaughter price.

3. Application of the model: An example

In order to illustrate the formulation and produced output of the model, the effect of the WSI on the replacement policy and the economic benefit of observing clinical signs of individual sows are presented. A hypothetical sow herd with two different risk levels for involuntary culling of sows are used for illustration.

3.1 Basic scenarios

The parameters of the model are chosen to be values that are considered common for Danish sow herds. Hence, settings from a commercial herd described by Kristensen and Søllested (2004b) (Herd A) are selected to represent a typical commercial Danish sow herd in this study. In the new version of the model, the feed price, piglet price, mating price, slaughter price per sow and disposal costs are updated (Table 2).

In order to adapt the model to the conditions of the commercial herd, information about the values of all variables in *H* and information about the observation policy are given. Two cases have been considered; one case where the values of all variables in the commercial herd define a high risk level (HR) herd of involuntary culling, and another case where the values define a low risk level (LR) herd of involuntary culling both in the gestation and lactation period. Hence, a HR herd is defined as a herd with a large herd size (>600 sows), using electronic sow feeding and with deep bedding in the pens. Contrary, a LR herd is a small herd (<400 sows) with feeding boxes and with no deep bedding in the pens.

The observation policy in both cases is assumed to be complete, meaning that all clinical examinations have been carried out in both the gestation and the lactation period. Therefore, no observation error is considered.

From the structure and probabilities of the resulting Bayesian networks, mean and standard deviation from both the gestation and lactation period are obtained (Table 3). In addition to these parameters, the necessary autocorrelation coefficients for the WSI are provided. From the gestation to the lactation period it is estimated to be ρ_{GLn} =0.1145 (see Appendix A), and it is assumed that the autocorrelation coefficient for the WSI from previous lactation period to current gestation period takes the same value. The WSI state variables are represented at $-2,...,0,...,+2$ levels so that k=2. The resulting state space is 130,628 states considering a maximum lifespan of 12 parities.

The parameters described above are given as input to the replacement model incorporating WSI information.

3.2 The effect of WSI on the replacement policy

The optimization of the model integrating the WSI is carried out for both the HR herd and the LR herd. Hence, optimal replacement policies maximizing average net returns over time are obtained.

For the HR herd, the optimal replacement policy implies no culling based on litter size and WSI before the 6^{th} parity. From the 6^{th} parity, the culling for low litter size or low WSI appears and increases through the 7^{th} parity. Culling actions are present in 40% and 71% of the defined states of the 6^{th} and the 7^{th} parity, respectively. All sows are culled independently of litter size after the 8^{th} parity. For the LR herd the same pattern is observed, except for the fact that the culling action is present in 39% and 72% of the defined states of the 6^{th} and the 7^{th} parity, respectively. The expected economic net returns obtained with optimal replacement policy is 2,472 DKK per sow per year for the HR herd, and 2,540 DKK per sow per year for the LR herd, thus being 3% higher in the LR herd compared to the HR herd. As the average herd size of a specialized Danish breeding farm is 1000 sows, this would represent 68,000 DKK/year in net returns.

The consequences of the LR herd and the HR herd are compared through technical and economic key figures calculated by Markov chain simulations (Table 4). The probability that a gilt inserted will end up being culled from the herd as dead/euthanized is 4% higher in the HR herd than the LR herd. As a consequence a lower average culling age is found in the HR herd compared to the LR herd (Table 4).
To illustrate the effect of the WSI on the replacement problem, the distribution of the WSI of voluntarily culled sows is computed (Figure 3). Hence, it appears that the WSI plays an important role when sows are selected for culling.

3.3 Economic value of observing clinical signs in sows

3.3.1 Definition of different scenarios

To investigate the economic benefit of clinical observations, 15 different scenarios are defined. Each scenario is represented by a specific observation policy and run for both a HR and a LR herd (Table 5).
Scenario 1 for a HR and LR herd (HR-1 and LR-1) is identical to the case presented previously, and will be used as the scenario of reference. Scenarios 2 to 7 represent scenarios where all clinical signs in the WSIs are observed, except: unwillingness to stand (scenario 2), lameness (scenario 3), vulva bite (scenario 4), body condition score (scenario 5), shoulder ulcer (scenario 6) and vulva color (scenario 7). In scenarios 8 to 13 only one clinical sign in the WSIs is observed: In scenario 8 only unwillingness to

stand is observed, in scenario 9 only lameness, in scenario 10 only vulva bite, in scenario 11 only body condition score, in scenario 12 only shoulder ulcer and in scenario 13 only vulva color is observed. Finally scenarios 14 and 15 represent scenarios where only clinical signs in the WSI of either the gestation (scenario 14) or lactation period (scenario 15) are observed.

3.3.2 Results from the example

The economic net returns of the set of scenarios are presented in Figure 4 as deviations from HR-1 and LR-1, respectively. The lowest reduction on the economic net returns is from scenario 2, meaning that if a farmer wants to reduce the number of clinical signs to observe, he should refrain from observing “unwillingness to stand” in pregnant sows. In contrast, the most expensive single clinical sign to leave out is “vulva bite” observed in pregnant sows (scenario 4). The same pattern is found in both the HR and the LR herd. However, the HR herd presents overall higher losses than the LR herd.

If a farmer avoids observing the clinical sign “shoulder ulcer” in lactating sows (scenario 6) in a HR herd, this will cause a higher economic reduction than if the farmer did not observe the clinical sign: Body condition score in lactating sows (scenario 5) (Figure 4).

If only one clinical sign can be observed, the clinical sign: “Vulva bite” (scenario 10) gives the lowest reduction in expected rewards compared to the base scenarios (where all signs are observed), while the highest reduction comes from observing the clinical sign: “unwillingness to stand” (scenario 8) for both a HR and a LR herd (Figure 4, 5). Thus, if a farmer only wants to observe one clinical sign, it should be "vulva bite" in pregnant sows.

Moreover, an analysis of scenarios 14 and 15 shows that observing clinical signs in the lactation period will cause a lower decrease in the net returns for both the HR and the LR herd compared to observing the set of clinical signs in the gestation period (Figure 4, 5). Hence, observing the set of clinical signs in the gestation period may allow for a better detection of weak sows than observing the set of clinical signs in the lactation period.

4. Sensitivity Analysis, autocorrelation

In order to test the effect of ρ_{LGn} of WSI on the voluntarily culling policy, the original value of autocorrelation is modified by ±50% in both types of herd (LR and HR). From the optimization model it is seen that changing the value of the autocorrelation has a limited impact on the results (Table 6). The size of the autocorrelation coefficient only marginally influences the distributions. Hence, the effect of a 50% of variation in the autocorrelation coefficient for the WSI from previous lactation period to current gestation period, ρ_{LGn} does not have any notable influence on the WSI estimates and economic rewards.

5. Discussion and Conclusion

In the dairy literature, several replacement models exit which incorporate information about the health of a cow in the model, and hence, make it influence the replacement decision (Stott and Kennedy, 1993; Houben et al., 1994; Gröhn et al., 2003; Heikkilä et al., 2008; Bar et al., 2009). Even though several computer-based simulation and optimization models have been developed for breeding and culling decision support on pig farms, the effects of the health status on optimal breeding and replacement policies have not been thoroughly studied.

In this study, a framework has been developed for incorporating the WSI information into an existing hierarchical Markov decision process model, which optimizes breeding and replacement decisions for pig herds. Combining information from several different clinical signs into one numerical value, the WSI, is an efficient tool, which makes it possible to handle incomplete observation strategies and evaluate their economic value. The herd level variables included in the Bayesian networks furthermore enable us to create herd specific models reflecting the risk factors of the individual herd. The presented framework could easily be extended to include more clinical observations without a combinatorial explosion of the size of the state space. It is therefore a powerful technique for dealing with the health status of individual animals in replacement models.

The numerical basis for establishing the WSI was systematic clinical examinations in 34 commercial sow herds combined with farmer records of all culled sows (including culling reasons). In a study like this it cannot be ruled out that the 34 farmers' culling decisions have been influenced by the very fact that they participated in the programme and, therefore, have been more careful than usual when sows were selected for culling or were euthanized. If such a carefulness has led to more or fewer involuntary cullings, the developed WSI will be biased accordingly.

The WSI is shown to have a high influence on the optimal replacement policy, allowing for a better economic classification of sow when taken the health status into account. It is also seen that the economic value of the WSI is higher in a HR than a LR herd. Hence, the total economic net returns are 3% higher in a low risk herd than in a high risk herd.

Observing the whole set of clinical signs in the WSIs allows for a better estimation of the WSI, and as a consequence a higher economic net returns. But in the case that the complete observation of all clinical signs cannot be observed, the most important sign from an economical point of view is “vulva bite” whereas the least important is “unwillingness to stand”. Using this information the farmer can better establish policies for observing clinical signs, and as a consequence, compare the economic benefit to the costs of observing the clinical signs.

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Appendix A

A.1 Presentation of the parameter estimates used for calculating the WSI probabilities.

Table A.1: Parameter estimates from the logistic analyses used to calculate the probabilities for the WSI for the pregnant sow. The parameter values form the numerical input to the Bayesian network shown in Figure 1.

Outcome variable	Explanatory variable	Parameter estimate
Involuntary culling	Intercept	-2.5718
	Factor: Lameness [a]	0.334
	Vulva bite	
	Yes	-0.7417
	No	0
Unwilling to stand	Intercept	-0.467
	Herd size	
	Small (< 400 sows)	-0.2852
	Average (400-600 sows)	-1.0944
	Large (> 600 sows)	0
	Feeding system	
	Electronic sow feeding	0.6146
	Individual based feeding	-0.5601
	Competition based feeding	0
Lameness	Intercept 1	-1.0925
	Intercept 2	-0.1875
	Herd size	
	Small (< 400 sows)	-0.7532
	Average (400-600 sows)	-0.9281
	Large (> 600 sows)	0
	Feeding system	
	Electronic sow feeding	-0.2754
	Individual based feeding	-0.7973
	Competition based feeding	0
Vulva bite	Intercept	-1.2666
	Deep bedding	
	No	-0.841
	Yes	0

a: The factor loading for the clinical sign: “lameness” was 0.51 and the factor loading for the clinical sign: “unwilling to stand” was 0.44

Table A.2: Parameter estimates from the logistic analyses used to calculate the probabilities for the WSI for the lactating sow. The parameter values form the numerical input to the Bayesian network shown in Figure 2.

Outcome variable	Explanatory variable	Parameter estimate
Involuntary culling	Intercept	-0.8098
	Shoulder ulcer	
	No scars or ulcers	-0.9763
	Scar	-1.101
	Ulcer	0
	Body condition score	
	Below average	1.3086
	Average	0.3006
	Above average	0
	Vulva colour	
	No	-2.6063
	Yes	0
Shoulder ulcer	Intercept 1	-2.0284
	Intercept 2	-0.9467
	Herd size	
	Small (< 400 sows)	0.259
	Average (400-600 sows)	1.2783
	Large (> 600 sows)	0
	Feeding system	
	Electronic sow feeding	0.8261
	Individual based feeding	-0.1208
	Competition based feeding	0
Herd size*feeding system	Small * Electronic sow feeding	-0.8214
	Small * Individual feeding	-04492
	Small * Competition feeding	0
	Average * Electronic sow feeding	-2.2932
	Average * Individual feeding	-0.5723
	Average * Competition feeding	0
	Large * Electronic sow feeding	0
	Large * Individual feeding	0
	Large * Competition feeding	0

The effect of the WSI on the conception rate was ψ=0.2224 (see Appendix B.5).

The dead fraction of gestation period parameter (ξ=0.61) was also estimated (see Appendix B.3.1).

Appendix B

B.1. Parameters needed for calculation of the transition probabilities

A list of the parameters, their symbols and sources are summarized in Table B.1.

Table B.1: Parameter needs for calculation of transition probabilities.

Parameter	Explanation	Source
μ_{hn}^{G}	Herd mean for parity *n* WSI in the gestation period	BN
μ_{hn}^{L}	Herd mean for parity *n* WSI in the lactation period	BN
σ_{Ghn}	Standard deviation of WSI between gestating sows	BN
σ_{Lhn}	Standard deviation of WSI between lactation sows	BN
σ_{Gn}	Standard deviation of the observation error of WSI for gestating sows when $E_h \subset S$	BN
σ_{Ln}	Standard deviation of the observation error of WSI for lactating sows when $E_h \subset S$	BN
ρ_{GLn}	Auto correlation between WSI in the gestation period and the subsequent lactation period of parity *n*.	DA
ρ_{LGn}	Auto correlation between WSI in the lactation period of parity *n* and the subsequent gestation period of parity *n*+1	AA
σ_{GOhn}	Total standard deviation of the observed WSI in the gestation period	ID
σ_{LOhn}	Total standard deviation of the observed WSI in the gestation period	ID
σ_{GLhn}	Standard deviation of the forecast error for WSI in the lactation period	ID
σ_{LGhn}	Standard deviation of the forecast error for WSI in the lactation period	ID
k	Number of WSI levels from $-k$ to k	DC
ξ	Fraction of death animals in gestation period	DA
ψ	Effect of WSI on conception rate	DA
w_{ni}^{G-}	Lower limit for i of estimated WSI in the gestation period of parity n (in herd h)	ID
w_{ni}^{G}	Mean for level i of estimated WSI in the gestation period of parity n (in herd h)	ID
w_{ni}^{G+}	Upper limit for level i of estimated WSI in the gestation period of parity n (in herd h)	ID
w_{ni}^{L-}	Lower limit for level i of estimated WSI in the lactation period of parity n (in herd h)	ID
w_{ni}^{L}	Mean limit for level i of estimated WSI in the lactation period of parity n (in herd h)	ID
w_{ni}^{L+}	Upper limit for level i of estimated WSI in the lactation period of parity n (in herd h)	ID

Source: BN=Bayesian network; DA=Data analysis; AA=Authors' assessment;

ID=Indirectly from the other values; DC=Decided.

B.2. A model for the transition probabilities

B.2.1 Levels of the state variables representing WSI

For each level $i \in \{-k,...,0,...,k\}$ of a WSI state variable, a lower limit w_i^-, a mean value w_i and an upper limit w_i^+ are calculated under the assumption that the WSI (on the logistic scale) is normally distributed. The values depend on stage of cycle and parity, but for simplicity further indexes are omitted. These values are determined under the standard assumption that all $2k+1$ levels have the same probability. It should be noticed, that $w_{-k}^- = -\infty$, $w_0 = 0$ and $w_k^+ = \infty$. For the determination of these level delimiters it must be remembered that the total variance of $\hat{w}_n^G$ is $\sigma_{GOhn}^2 = \sigma_{Ghn}^2 + \sigma_{Gn}^2$ and accordingly for $\hat{w}_n^L$.

In the following, a level i is also identified by its mean, w_i.

B.2.2 The initial distribution of $\hat{w}_n^G$ for a gilt

Since the WSI of gilt is the first one observed, no prior knowledge is available, and

$$P(\hat{w}_{1i}^G) = \Phi\left(\frac{w_{1i}^{G+}}{\sigma_{GOhn}}\right) - \Phi\left(\frac{w_{1i}^{G-}}{\sigma_{GOhn}}\right) \qquad (6)$$

where Φ is the distribution function of the standard normal distribution.

B.2.3 The conditional distribution of $\hat{w}_n^L$ given $\hat{w}_n^G$ for a gilt or sow

It is assumed that $P(\hat{w}_n^L \mid \hat{w}_{n-1}^L, \hat{w}_n^G) = P(\hat{w}_n^L \mid \hat{w}_n^G)$ even for $n > 1$.

A simple first order auto regressive model is assumed for the WSI. Let ρ_{GLn} be the correlation coefficient between the WSI in the gestation period and the subsequent lactation period.

Thus,

$$w_n^L = \mu_{hn}^L + \frac{\rho_{GLn}\sigma_{Lhn}}{\sigma_{Ghn}}(w_n^G - \mu_{hn}^G) + \eta_{GLn}, \qquad (7)$$

where $\eta_{GLn} \sim N(0,(1-\rho_{GLn}^2)\rho_{Lhn}^2)$. Recalling from Eq.(5) that, for a given sow with observed clinical signs.

$$w_n^G = \mu_{hn}^G + \hat{w}_n^G + \varepsilon_{Gn} \qquad (8)$$

where $\varepsilon_{Gn} \sim N(0,\sigma_{Gn}^2)$. Accordingly,

$$w_n^L = \mu_{hn}^L + \hat{w}_n^L + \varepsilon_{Ln} \qquad (9)$$

where $\varepsilon_{Ln} \sim N(0,\sigma_{Ln}^2)$. Substituting (8) and (9) into (7) and reducing gives us,

$$\hat{w}_n^L + \varepsilon_{Ln} = \frac{\rho_{GLn}\sigma_{Lhn}}{\sigma_{Ghn}}(\hat{w}_n^G - \varepsilon_{Gn}) + \eta_{GLn}$$

By further reduction the following expression is obtained

$$\hat{w}_n^L = \frac{\rho_{GLn}\sigma_{Lhn}}{\sigma_{Ghn}}\hat{w}_n^G + \varepsilon_{GLn} \qquad (10)$$

where $\varepsilon_{GLn} = \eta_{GLn} + \frac{\rho_{GLn}\sigma_{Lhn}}{\sigma_{Ghn}}\varepsilon_{Gn} - \varepsilon_{Ln}$ implying that $\varepsilon_{GLn} \sim N(0,\sigma_{GLhn}^2)$, where

$$\sigma_{GLhn}^{2} = \left(1 + \rho_{GLn}^{2}\left(\frac{\sigma_{Gn}^{2}}{\sigma_{Ghn}^{2}} - 1\right)\right)\sigma_{Lhn}^{2} + \sigma_{Ln}^{2} \tag{11}$$

It is now possible to specify the transition probabilities from WSI = i in the gestation period to WSI= j in the lactation period:

$$P(\hat{w}_{nj}^{L} \mid \hat{w}_{ni}^{G}) = \Phi\left(\frac{w_{nj}^{L+} - \frac{\rho_{GLn}\sigma_{Lhn}}{\sigma_{Ghn}}\hat{w}_{ni}^{G}}{\sigma_{GLhn}}\right) - \Phi\left(\frac{w_{nj}^{L-} - \frac{\rho_{GLn}\sigma_{Lhn}}{\sigma_{Ghn}}\hat{w}_{ni}^{G}}{\sigma_{GLhn}}\right) \tag{12}$$

B.2.4 The conditional distribution of $\hat{w}_{n}^{G}$ given $\hat{w}_{n-1}^{L}$ for a gilt or sow

Analogously, the transition probabilities from WSI = i in the lactation period to WSI= j in the next gestation period become:

that

$$P(\hat{w}_{nj}^{G} \mid \hat{w}_{n-1,i}^{L}) = \Phi\left(\frac{w_{nj}^{G+} - \frac{\rho_{LGn}\sigma_{Ghn}}{\sigma_{Lhn}}\hat{w}_{n-1,i}^{L}}{\sigma_{LGhn}}\right) - \Phi\left(\frac{w_{nj}^{G-} - \frac{\rho_{LGn}\sigma_{Ghn}}{\sigma_{Lhn}}\hat{w}_{n-1,i}^{L}}{\sigma_{LGhn}}\right) \tag{13}$$

B.3. Transition probabilities to State “Dead”

B.3.1. Gestation Period

The probability follows directly from the estimated WSI which is considered to be a direct estimate for the involuntarily culling (both death and send to slaughter probability) on the logistic scale. Since it has defined that low relative values of WSI means a weak sow (and high values accordingly refer to strong sows) it is defined the corresponding logistic value y_{ni}^{G} for a parity n sow with WSI index level i in the gestation period as

$$y_{ni}^{G} = \mu_{hn}^{G} - \hat{w}_{ni}^{G} \qquad (14)$$

Thus, the probability of involuntarily culling becomes

$$P(Dead, Diseased \mid \hat{w}_{ni}^{G}) = \frac{1}{e^{-y_{ni}^{G}} + 1} \qquad (15)$$

The data analysis performed by Jensen et al. (2009) showed no correlation between the fraction of dead/euthanized sows among the involuntarily culled sows. A fixed fraction of $\xi = 0.61$ was found.

Thus, the probability of death becomes

$$P(Dead \mid \hat{w}_{ni}^{G}) = \xi \times P(Dead, Diseased \mid \hat{w}_{ni}^{G}) \qquad (16)$$

B.3.2. Lactation Period

The probability follows directly from the estimated WSI which is considered to be a direct estimate for the death probability on the logistic scale. Since it has defined that low relative values of WSI means a weak sow (and high values accordingly refer to strong sows) it is defined the corresponding logistic value y_{ni}^{L} for a parity n sow with WSI index level i in the lactation period as

$$y_{ni}^{L} = \mu_{hn}^{L} - \hat{w}_{ni}^{L} \qquad (17)$$

Thus, the probability of death becomes

$$P(Dead \mid \hat{w}_{ni}^{L}) = \frac{1}{e^{-y_{ni}^{L}} + 1} \qquad (18)$$

B.4. Transition probabilities to State "Diseased"

For the gestation period, the diseased sows are just the involuntarily culled sows that are not dead/euthanized. Thus, $P(Diseased \mid \hat{w}_{ni}^{G}) = \quad (1-\xi) \times P(Dead\,,\, Diseased \mid \hat{w}_{ni}^{G})$ **.**

(16)

In the lactation period this state in the version of the replacement model as was described by Kristensen and Søllested (2004a,b) corresponds to a sow being involuntarily culled either because of death/euthanization or premature slaughtering. However in the current model with WSI this state only corresponds to premature slaughtering, since the state "Dead" corresponds to death/euthanization.

Thus, the probability of entering the "Diseased" state must be reduced by the probability of entering the "Dead" state. In the current version of the model, the probability of involuntary culling only depends on parity and stage (i.e. mating, gestation, lactation), but it seems more logical to let the total probability of involuntary culling ("Diseased" and "Dead") depend on the WSI.

The applied procedure for the gestation stage is as follows:

- Denote the current probability of involuntary culling for a parity n sow in the gestation period as q_n^L and the corresponding logistic value as z_n^L, i.e.,

  $$z_n^L = \ln \frac{q_n^L}{1 - q_n^L}.$$

- Define q_n^L as the probability of involuntary culling for a sow in WSI level $i = 0$ (i.e. a sow with an average WSI).

- For other WSI levels $i \in \{-k,...,0,...,k\}$, the logistic value of the probability of involuntary culling is adjusted by the numerical value of the WSI level, i.e. the logistic value z_{ni}^L for WSI level i becomes

$$z_{ni}^L = z_n^L - \hat{w}_{ni}^L \qquad (16)$$

Thus the probability of involuntary culling becomes

$$P(Dead, Diseased \mid \hat{w}_{ni}^L) = \frac{1}{e^{-z_{ni}^L} + 1} \qquad (17)$$

Finally, the probability of entering the "Diseased" state becomes:

$$P(Diseased \mid \hat{w}_{ni}^G) = P(Dead, Diseased \mid \hat{w}_{ni}^G) - P(Dead \mid \hat{w}_{ni}^G) \qquad (18)$$

B.5. Adjustment of the conception rate

The conception rate as defined in the model by Kristensen and Søllested (2004a,b) is in the present extended model adjusted for the influence of the WSI of the most recent lactation period. The procedure is that for WSI level *i* = 0 (an average sow), the original conception rate is used. Denote as y_n the logistic transform of this conception rate for parity *n*. For other WSI levels, the logistic transform is adjusted linearly in the WSI:

$$y_{ni} = y_n + \psi \cdot \hat{w}_{ni}^G,$$

where the coefficient in the data analysis by Jensen et al. (2009) was estimated as $\psi = 0.2224$. Finally, the adjusted conception rate is calculated as

$$(conception \quad rate | w_{ni}^G) = \frac{1}{e^{-y_{ni}} + 1}.$$