OPTIMAL LENGTH OF LEYS IN AN AREA WITH WINTER DAMAGE PROBLEMS

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ABSTRACT

A model for investigation of the optimal economic life cycle of grass leys with winter damage problems in northern Norway and the threshold of winter damage before it is profitable to re-seed grasses is presented. A 2-level hierarchic Markov process has been constructed using the MLHMP software. The model takes uncertainty concerning the yield potential, damage estimation and weather dependent random fluctuations into account. A Kalman filter technique is used for updating of knowledge on yield potential and damage level.

1. INTRODUCTION

Grass production is the main agricultural land use in many parts of Norway. In northern Norway, i.e., the three counties of Nordland, Troms and Finnmark, as much as 93 per cent of agricultural land in use was under grass in 1999 (Anonymous, 2000). The profitability of grassland depends strongly on choice of the appropriate length of ley periods (Hegrenes, 1991). Winter damage of the ley is one reason why the grass leys have to be re-seeded. In the years 1975, 1978, 1985, 1995, and 1998 the leys were severely damaged on many farms in northern Norway. Andersen (1960) reported a relatively high frequency of winter damage in some regions in northern Norway in the period 1922-59. Clearly, winter damage to grassland is a significant hazard in this area.

Young grass leys normally have higher yields than older leys (Nesheim, 1986). Therefore, even under 'normal' as well as under winter damage conditions, it might be beneficial to plough the fields and re-seed grasses. In this article we examine the replacement of leys from the farmer's point of view.

Deciding whether to re-establish grass leys is a typical replacement problem. Such problems are effectively handled by dynamic programming. For surveys, reference is made to Kennedy

(1986); Kristensen (1994). A farmer who cultivates grassland is always faced with uncertainty. For instance, the farmer will not know for sure what yields he will obtain in future. This uncertainty concerning future yields has several different sources:

- Even though the farmer knows the yield history of the field, there is still some uncertainty concerning the true yield potential.
- In spring, the farmer may inspect the field and visually estimate the extent of a possible winter damage. There is, however, a considerable uncertainty concerning the visual estimate.
- Weather conditions may influence the year-to-year yields considerably.

It is the aim of this study to construct a model taking all the uncertainties mentioned into consideration. A 2-level hierarchic Markov process (Kristensen and Jørgensen, 2000) is build using the MLHMP software (Kristensen, 2002). A Kalman filter model (West and Harrison, 1997) is used for representing and updating knowledge about the yield potential of the field.

2. A GRASSLAND YIELD MODEL

2.1. General model

Grassland yields are in general assumed to follow a quadratic function

$$y(n) = \begin{cases} y_0 - \frac{2(y_0 - \overline{y})n}{\overline{n}} + \frac{(y_0 - \overline{y})n^2}{\overline{n}^2}, & n \leq \overline{n} \\ \overline{y}, & n > \overline{n} \end{cases} \tag{1}$$

where $y(n)$ is the expected yield for year $n+1$ since re-seeding (i.e. $y(0)$ is the expected yield for the first year after re-seeding). The parameters y_0, $\overline{y}$ and $\overline{n}$ are the initial yield, the minimum yield and the year of minimum yield, respectively. Having reached the minimum yield, it is assumed to remain at this minimum value.

In order to implement the quadratic yield curve on a particular field, we need to know the initial yield y_0, the minimum yield $\overline{y}$ and the age for minimum yield $\overline{n}$. Those parameters are rather concrete and the farmer and his advisor will probably have a rather precise idea of the values for a field. We shall therefore assume that those parameter values are known. When we apply the model (1) on a particular field i, we shall index the symbols by i (e.g. $y_i(n)$) in order to indicate that we refer to that particular field.

Nevertheless, the actual yield of a field will vary at random from year to year, and from ley to ley. We may include those aspects by the following extension

$$Y_{ij}(n) = y_i(n)L'_j + \epsilon_n \tag{2}$$

where $Y_{ij}(n)$ is the observed actual yield at year n from field i, ley j, $L'_j \sim N(1, \sigma_L^2)$ is the random multiplicative effect of ley, and $\epsilon_n \sim N(0, \sigma^2)$ is a random year-to-year fluctuation. In other words, if $L'_j = 1.05$, it means that the actual expected yield of the ley is 5% higher than assumed. The effect of L'_j is partially a result of the climatic and other conditions around the year of re-seeding and partially we may interpret it as uncertainty regarding our choice of field characteristics expressed by the yield estimates (y_0, $\overline{y}$) and year of minimum yield ($\overline{n}$).

The yield of a ley may be observed at the end of the growing season, but already at the beginning of the season in spring, the farmer may inspect the field and estimate the extend of a possible winter damage. The usual way of numerically expressing the damage is in the form of ratio α_n estimating the relative reduction in yield caused by the damage. If we further extend our yield model with this effect, we obtain the following model

$$Y_{ij}(n) = y_i(n)L_j(n) + \epsilon_n \tag{3}$$

where now $L_j(n)$ expresses the relative yield potential of the ley corrected for the effect of one or several winter damages since re-seeding. We shall assume that the farmer is able to give an unbiased estimate, α_n of the damage, but with an uncertainty expressed by the value of σ_D such that

$$(L_j(n) \mid L'_j) \sim \mathrm{N}((1-\alpha_n)L'_j, \sigma_D^2). \tag{4}$$

2.2. Field and ley level

In the model represented by Eqs. (3) and (4) only the resulting yield $Y_{ij}(n)$ and the estimated relative size α_n are observable. In the following we shall only refer to one particular field and ley, and we may therefore skip the indexes for field and ley:

$$Y(n) = y(n)L(n) + \epsilon_n \tag{5}$$

If we apply the Kalman filter technique as described by West and Harrison (1997) we may interpret the yield model (5) as the observation equation. The corresponding system equation of the Kalman filter is

$$L(n) = F_n L(n-1) + e_n \tag{6}$$

where

$$F_n = \begin{cases} 1, & no\ damage\ neither\ previos\ nor\ present\ year \\ \frac{1-\beta\alpha_n}{1-\alpha_n}, & damage\ last\ year,\ no\ new\ damage \\ 1-\alpha_n, & damage\ this\ year \end{cases} \tag{7}$$

and $e_n \sim \mathrm{N}(0, \sigma_{Sn}^2)$ is a residual reflecting the system variance defined as

$$\sigma_{Sn} = \begin{cases} 0, & no\ damage\ this\ year \\ \sigma_D^2, & damage\ this\ year \end{cases} \tag{8}$$

The interpretation of Eqs. (6) - (8) is that the yield potential $L(n)$ is assumed constant over time as long as no damage has occurred or if the latest damage is at least two years old. If a damage occurred last year, the effect of the damage is assumed to have decreased to a level defined by β due to partial recovery of the field. In years where a new winter damage is observed, α_n must express the size of the relative damage, and the loss is expressed as the expected value plus a random term of which the variance depends on the uncertainty (σ_D^2) of the farmer's estimate.

It is obvious that the true value of $L(n)$ is important for the re-seeding decision since it represents a permanent trait of the ley and the size of a winter damage. Since we are not able to observe the true value, we have to estimate it using Bayesian updating as discussed by Kristensen (1993).

The prior estimate of $L(n)$ before we have observed any values of $Y(n)$ is 1. We may express this as $\hat{L}_0 = 1$. In general, we shall denote the current estimate of $L(n)$ after the observation of n years of yields $\hat{L}_n$ and accordingly for other parameters. Now, assume that we know the estimates $\hat{L}_n$, and furthermore observe the yield $Y(n+1)$ of year $n+1$. We may then update our belief in the true value of $L(n+1)$ using the following relation taken from West and Harrison (1997):

$$\begin{array}{lcl} \hat{L}_{n+1} & = & F_{n+1}\hat{L}_n + A_{n+1}(Y(n+1) - y(n+1)F_{n+1}\hat{L}_n) \\ A_{n+1} & = & R_{n+1}y(n+1)V_{n+1}^{-1} \\ R_{n+1} & = & F_{n+1}^2 C_n + \sigma_{Sn}^2 \\ V_{n+1} & = & (y(n+1))^2 R_{n+1} + \sigma^2 \\ C_{n+1} & = & R_{n+1} - A_{n+1}^2 V_{n+1} \\ \hat{L}_0 & = & 1 \\ C_0 & = & \sigma_L^2 \end{array} \tag{9}$$

The value C_n is the variance of $\hat{L}_n$, and $\hat{L}_0$ and C_0 are the initial values of $\hat{L}_n$ and C_n for $n = 0$, i.e. before the first yield result is observed. The variance components σ_L^2 and σ^2 from Eq. (9) must be estimated from yield data.

In the prediction of the next yield we want to be as precise as possible using all previous information (i.e. all previous yield observations of the ley). The benefit of the Kalman filter technique combined with the proposed yield model (3) is that the expected value of $Y(n+1)$ given all previous information only depends on $\hat{L}_n$. The conditionally expected value of $Y(n+1)$ is simply

$$\mathrm{E}(Y(n+1)|Y(n)\ldots,Y(0),\hat{L}_0,C_0) = \mathrm{E}(Y(n+1)|\hat{L}_n) = y(n+1)F_{n+1}\hat{L}_n, \tag{10}$$

and the conditional variance is simply V_{n+1} as defined in Eq. (9). It should be noticed that the conditional variance V_{n+1} varies with n, but independently of the observations of $Y(n)$. From Eqs. (7) to (9), however, we conclude that we need to know the precise damage history $\alpha_1,\ldots,\alpha_{n+1}$ in order to know the precise values of V_{n+1} and C_n. Given this damage history, it is possible to calculate the sequence $V_1,\ldots,V_n$ for any n without observing $Y(1),\ldots,Y(n)$. The conclusion is that in order to predict the next yield result we only need to keep the most recent value of $\hat{L}_n$.

The parameters needed for a full specification of the yield model are summarized in Table 1.

Table 1: Parameter needs for the yield model

Parameter	Explanation
y_0	First full year yield of the field
$\overline{y}$	Minimun yield of the field
$\overline{n}$	Year of minimum yield
σ	Random year-to-year standard deviation
σ_L	Standard deviation of multiplicative ley effect
σ_D	Standard deviation for estimated winter damage
β	Relative decrease in size of winter damange

3. OPTIMIZATION MODEL

3.1. Model structure

The tool to be used in this re-seeding model is a multi-level hierarchic Markov process as described by Kristensen and Jørgensen (2000). A multi-level hierarchic Markov process has an ordinary infinite time Markov decision process running at the *founder level*. For each combination of state and action, a stage of the founder may be represented by a *child process*, which in turn is an ordinary finite time Markov decision process. For each combination of state and action of the child process, the stage might again be extended to a (grand-)child level etc. In this case, however, we use a model with only two levels as follows:

Founder process Infinite time horizon.

Stage Stage length is equal to the life time of one particular ley.

State space Only one dummy state is defined.

Action space Only one dummy action is defined.

Child level 1 Finite time.

Stage Stage length is equal to a year.

State space The state is defined by the values of the following *state variables* at stage n:

Estimated ley potential The value of $\hat{L}_{n-1}$ - the estimated potential after $n-1$ observations (21 classes).

Variance of estimate The value of C_{n-1} - variance of the estimated potential after $n-1$ observations (21 classes).

Winter damage The value of α_n. Six levels are considered: α_n = 0.00, 0.20, 0.30, 0.40, 0.50 or 0.75.

Age of damage Two levels: “This year”, “Previous year”.

If the winter damage is 75%, all other state variables are ignored, and the decision “Re-seed” is chosen without further consideration. The total number of states equals 3970 per stage.

Action space Two options: “Keep” or “Re-seed”. If the winter damage is 75% only the decision “Re-seed” is available.

The total number of state combinations is around 75,000. The model was constructed as a plug-in to the MLHMP software[1] (Kristensen, 2002).

3.2. Parameters

The output, $m_i^d(n)$, of a stage under the action “Keep” is simply the expected yield calculated according to Eq. (10), and under the action “Re-seed” it is 0. As concerns stage length, it is 1 year if the ley is kept and 0 if re-seeded. When we refer to state i at stage n, we mean the state $i = (\hat{L}_{n-1}, \alpha_n, C_{n-1}, a_n)$ where a_n is the age of the damage (if any).

The rewards, $r_i^d(n)$, of the model are defined as the expected net revenues in a particular state under a given action. The expected net revenue is calculated as the value of the coarse fodder produced minus the variable costs which are assumed to depend on expected yield according to a quadratic function. For the decision “Re-seed” ($d = 1$), the reward is simply zero since all costs related to re-seeding are included at stage 0 of the new ley.

The transition probabilities, $p_{ij}^d(n)$, define the probabilities of state transitions from state $i = (\hat{L}_{n-1}, \alpha_n, C_{n-1}, a_n)$ at stage n to state $j = (\hat{L}_n, \alpha_{n+1}, C_n, a_{n+1})$ at stage $n+1$. The uncertainty concerning the transition is governed by two random events: The (later) observed yield $Y(n)$ of this year which is normally distributed and the observed level of winter damage next year which has a simple multinomial distribution over the values 20%, 30%, 40%, 50% and 75%. All other effects are deterministic. Based on the two known random distributions and the deterministic transitions, the over-all transition probabilities are calculated analytically.

4. DISCUSSION

Even though the model with its only 75,000 states is of modest size compared to other models published it has nevertheless turn out to be a challenge to the MLHMP software even though it indeed is tractable. In particular the model generation process is rather time consuming whereas the optimization on the other hand is very fast. The main reason for the slow model generation compared for instance to the much larger model presented by Nielsen and Kristensen (2002) is that the decomposition of the state space in this case is limited to the stages representing the age of the lay. The state space at the child level is therefore rather large with 3970 states per stage. In the steer production model (Nielsen and Kristensen, 2002) a far better decomposition resulting in 4 levels was possible. The benefit of using a hierarchic model was therefore higher in that case.

It should be noticed that a main reason for the poor decomposition of the state space in this case is the state variable C_n representing the variance of the estimated yield potential. If that variable was omitted the state space could be reduced with a factor 21 to only 190 states. That

[1]The MLHMP software and the plug-in constructed for this model are available for download at http://www.prodstyr.ihh.kvl.dk/software/mlhmp.html.

would be possible, if the farmer was able to observe the damage level with certainty (or if the uncertainty concerning the assessment was ignored).

The poor decomposition of the state space makes this model an obvious candidate for representation as a mixed Markov LIMID model as discussed by Nilsson and Kristensen (2002), since in a LIMID the state space is represented by individual state variables in stead of an aggregated state space formed as a cartesian product of the individual state variables.

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