A SOW REPLACEMENT MODEL USING BAYESIAN UPDATING IN A 3-LEVEL HIERARCHIC MARKOV PROCESS *

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ABSTRACT

Recent methodological improvements in replacement models comprising multi-level hierarchical Markov prosesses and Bayesian updating have hardly been implemented in any replacement model and the aim of this study is to present a sow replacement model that really uses these methodological improvements. The model is developed as a prototype for use under practical conditions. The application of the model is demonstrated using data from two commercial Danish sow herds. It is concluded that the Bayesian updating technique and the hierarchical structure decrease the size of the state space dramatically. Since parameter estimates vary considerably among herds it is concluded that decision support concerning sow replacement only makes sense with parameters estimated at herd level.

1. INTRODUCTION

Several replacement models have been presented in literature and various methodological improvements have been developed. These improvements comprise multi-level hierarchical Markov processes with decisions on multiple time scales, Bayesian updating and herd specific parameter estimation. In spite of the theoretical development these methodological improvements have not yet been applied to models regarding sow replacement.

In the replacement model a biological as well as an optimization model are needed. The biological model is used to predict the future performance of the individual sows in respect to their productivity and the optimization model optimizes the replacement decision in respect to the parameter estimates given by the biological model. Since in particular litter size parameters

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and conception rates vary considerably between herds (Jørgensen, 1992) the need for parameter estimation at herd level seems important.

The sow replacement model is implemented as a prototype for use under practical conditions and described in detail by Kristensen and Søllested (2001a,b). In order to illustrate the applicational potential of the model an example of using the model with data from two commercial Danish sow herds is given. A recently released free standard software called MLHMP *(Multi-Level Hierarchical Markov Processes)* (Kristensen, 2000) has been used to implement the model.

2. BIOLOGICAL AND OPTIMIZATION MODELS

The core element of any replacement model is *prediction*. Therefore the purpose of the underlying biological model is to provide us with as precise predictions as possible given all information available about a sow. However predicting the future might be related to some uncertainty and the major task of the biological sub-models is to minimize this uncertainty.

The most important product from a sow herd is of course piglets, so predictions of future litter sizes and dates of farrowing are essential. We therefore need sub-models for litter size and conception rates. In order to be able to predict the economic revenue from piglets sold and sows slaughtered we need sub-models for piglet mortality and weight of sows. Furthermore we need sub-models for feed intake and involuntary culling.

One of the major problems in the biological sub-models is a markedly herd effect. These herd effects cover among others the effects of different management strategies and routines. One way to handle the herd effects might be to estimate parameters on herd level. However less information is often available at herd level and for some herds information of old sows might not exist due to hard culling of sows (Toft and Jørgensen, 2002). If this fact is ignored the replacement decisions will be erroneous in that it becomes uncertain or even impossible to estimate the effects of i.e. large parity sows.

In order to handle this complex situation Toft and Jørgensen (2002) combined information from 30 herds in estimation of herd specific litter size parameters by allowing the herd specific estimates to be based on herd information as well as information from the entire population (30 herds). The less information available of a single herd the greater influence of the population information on parameter estimates. By introduction of the Bayesian updating of litter size parameters, Toft and Jørgensen (2002) demonstrated the possibility to obtain reasonable herd specific estimates although herd information was limited. By applying the litter size model of Toft and Jørgensen (2002) the need of herd specific estimates in replacement decisions as proposed by Jørgensen (1992) seems to be fulfilled.

However the litter size sub-model is only one of several sub-models in the presented sow replacement model and for the rest of the sub-models different approaches have been used. For all of these sub-models estimation has been made either directly on population information or on herd information. A detailed description of the biological sub-models is given by Kristensen and Søllested (2001a).

The optimization models used in this replacement model is a multi-level hierarchic Markov process as described by Kristensen and Jørgensen (2000). A multi-level hierarchic Markov process has an ordinary infinite time Markov decision process running at the *founder level*. For each combination of state and action, a stage of the founder may be represented by a *child process*, which in turn is an ordinary finite time Markov decision process. For each combination of state and action of the child process, the stage may again be extended to a (grand-)child level.

As described by Kristensen and Søllested (2001a), the litter size (total number of piglets born) at parity n of a sow is described as

$$Y_n = \mu_n + M(n) + \epsilon_n, \tag{1}$$

and the estimate $\hat{M}(n)$ are updated at sow level each time a new litter size is observed.

The model presented in this paper has 3 levels defined as follows:

Founder process Infinite time horizon

- **Stage** Stage length is equal to the life span of a sow in the herd
- **State space** Only one dummy state is defined
- **Action space** Only one dummy action is defined

Child level 1 Finite time

- **Stage** Stage length is equal to a reproductive cycle from weaning to weaning. Stage number equals parity.
- **State space** Depends on parity:
  - **Parity 1** Only one dummy state is defined.
  - **Parity 2** Litter size of parity 1. An additional state representing a culled sow is added. The number of states equals $20 + 1 = 21$.
  - **Parity** $n > 2$ Estimated serially correlated effect $\hat{M}(n)$ (21 classes). An additional state representing a culled sow is added. The number of states equals $21 + 1 = 22$.
- **Action space** Mating method: 2 options that for instance represent "Natural mating" and "Artificial Insemination". Each action has its own effect on conception rate, price of mating and price of the resulting piglets.

Child level 2 Finite time

- **Stage** Stage length is equal to the duration of “Mating” (stage 1), “Gestation” (stage 2) or “Suckling” (stage 3).
- **State space** Depends on stage:
  - **Stage 1, “Mating”** Two states reflecting health status: “Healthy” and “Diseased”.
  - **Stage 2, “Gestation”** Three states: “Pregnant”, “Infertile” and “Diseased”.

Stage 3, "Suckling" Litter size (total number born), present parity. An additional state representing a diseased sow is added. The size of the state space is $20 + 1 = 21$.

Action space Depends on stage:

Stage 1, "Mating" Mating policy: Allow $1, \ldots, 5$ matings before culling for infertility if the sow is "Healthy". If the sow is "Diseased", only one dummy action is defined.

Stage 2, "Gestation" Only one dummy action is defined.

Stage 3, "Suckling" Two actions defined: "Keep the sow" and "Replace the sow at weaning".

The consequence of a sow entering one of the "Diseased" states of child level 2 is that it is culled at once, and that the process terminates as described by Kristensen (2000). The process at child level 1 then enters the "Culled" state with probability 1.

In general, parameters are defined at child level 2. At founder level and child level 1, the parameters are calculated from the corresponding child processes as described by Kristensen and Jørgensen (2000).

Transition from one stage to another is determined by transition probabilities given by the biological sub-models of litter-size and conception rates.

The rewards of the model are equal to the expected economic net revenue of a sow during the stage in question given all information available (i.e. the stage, state and action of child level 2 and both ancestral levels). The economic net returns are calculated as revenues from sold piglets and culled sows minus feed costs, mating costs and costs of replacement gilts.

3. RESULTS

Optimal policies maximizing average net returns over time for the two herds were calculated. In Table 1, the consequences of the optimal policies are illustrated by selected technical and economical results. The figures of the table are calculated by means of the Markov chain simulation facility of the MLHMP software. They are all based on mating method 1.

In words, the optimal replacement policy for Herd A implies no culling based on litter size before parity 5. From parity 5 through 10 the culling for low litter sizes increases and all sows are culled after the 11th parity independently of litter size results obtained. For Herd B, the same pattern is found, but no culling for low litter size should be carried out before parity 6. This is in agreement with the higher average culling age seen in Table 1 for Herd B. Even though there is no direct culling for litter size before parity 5 and 6, respectively, there may be an indirect effect trough the number of matings allowed before culling for infertility. In Figure 1 the optimal mating policies in the two herds are shown. For the same litter size potential and parity, typically 1 additional mating is optimal in Herd B compared to A.

In Figure 2 the optimal age distributions of the two herds are compared to the observed age distributions. As it appears, Herd A seems to use a replacement policy that is very close to the optimal. Herd B, on the other hand, uses a far too high culling rate.

Table 1: Technical and economical results for the two herds under optimal policies.

Key figure	Herd A	Herd B
Net returns, DKK per sow per year	3526	4128
Net returns, DKK per piglet weaned	168.99	176.77
Piglets weaned per sow per year	20.87	23.35
Piglets weaned per litter	8.80	9.95
Average age (parity) at culling	5.46	6.33
Voluntary annual culling rate	26%	14%
Involuntary annual culling rate	17%	23%

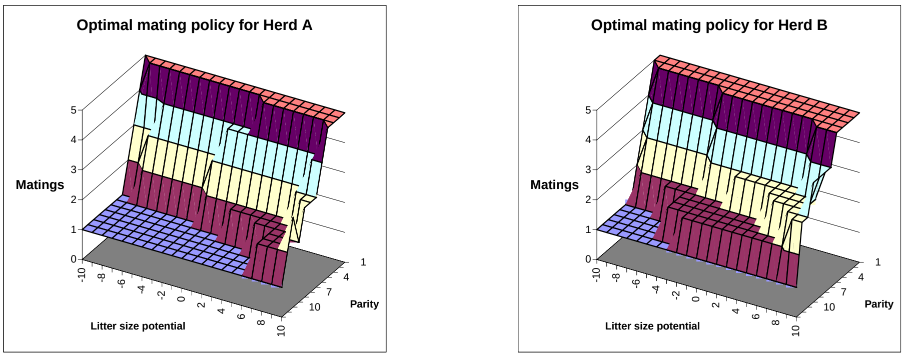

Figure 1: Optimal mating policies depending on litter size potential and parity for the two herds.

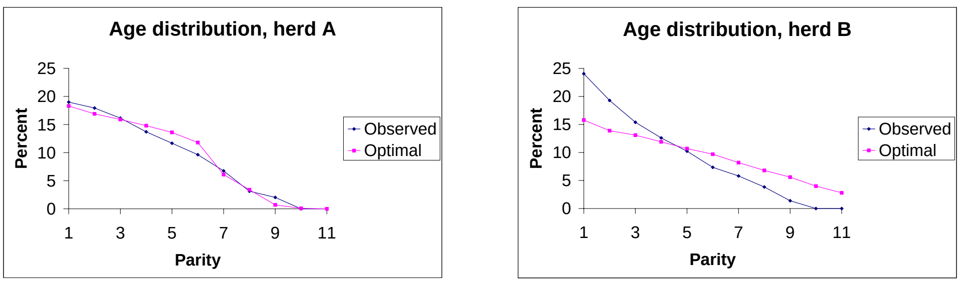

Figure 2: Optimal age distributions of the two herds compared to the observed distributions.

Also the mating method is optimized in the model. When the relative effect of method 2 was set to 0.9, it was generally preferred in both herds. At a relative effect of 0.85 method 2 was preferred to the sows having the highest expected litter size, and at a relative effect of 0.80, method 1 was generally preferred. In other words, if the relative effect of method 2 is higher than 85%, the higher value of the piglets will compensate for the higher mating costs.

The parameter sets for the two herds differ considerably in many respects. Concerning litter sizes, and piglet mortality, Herd B performs far better than Herd A, but when it comes to conception rates, Herd A has the best performance. From the Markov chain simulation results of Table 1 it is evident that the better litter size results (measured as weaned piglets) of Herd B leads to far better economical results under an optimal policy despite the lower conception rates.

4. DISCUSSION

The presented sow replacement model has several features aiming at reducing the size of the state space. First of all there is the Bayesian updating technique used with the litter size model. Using this technique enables us to take *all* previous litter size information into account just by including two variables representing the current estimates of the sow specific litter size parameters in the state space of the model. As opposed to the Bayesian updating method the traditional way of including information about previous litter size results is to define the state space as the cartesian product of all previous litter size results. The strength of the Bayesian updating technique is that it is possible to summarize the effect of all previous parities through only two variables *without loss of information* given the litter size model used. Thus the state space concerning litter size only contains approximately 400 states compared to 1.7×10^8 if the same information should be represented by traditional methods.

An other feature aiming at reducing the size of the state space is the multi-level hierarchical formulation allowing for decisions on multiple time scales. In this model we integrate decisions concerning mating method and replacement. Since the mating method influences the future value of the piglets, the information about method has to be kept until weaning. The traditional way of keeping such information would be to define a state variable remembering the decision so that the size of the state space would be doubled. By using the hierarchical technique, we completely avoid this explosion in state space.

As it has been loosely illustrated by the two herds used in this study, the biological parameters differ considerably among herds. This observation is confirmed by the results of Toft and Jørgensen (2002) who estimated litter size parameters and dropout rates in more than 40 Danish sow herds. Since the genetic properties only varies slightly among herds, the explanation must be found in the characteristics of the production systems and the general levels of management. Because of this variation it seems obvious that decision support concerning sow replacement must be based on herd specific estimates in order to make sense. The estimation technique presented by Toft and Jørgensen (2002) enables us to obtain such herd specific estimates.

The multi-level hierarchical Markov process is a very flexible tool. At the most detailed level (child level 2) the stage length equals the duration of the mating period, the gestation period and the suckling period, respectively. In case a more detailed modeling of any of these periods is desired, the model may be extended by a child level 3 in one or more stages of child level 2.

Similarly, if we wish to evaluate decisions with a longer time horizon than considered here, we may include actions at founder level.

The MLHMP software used in this study has turned out to be an important shortcut in the development of working prototypes. Even though the process still demands some programming (in this case in Java) in the creation of the plug-in, it is certainly an advantage that the basic algorithms used by multi-level hierarchical Markov processes have been programmed once for all.

5. CONCLUSION

The model presented demonstrates that proper use of Bayesian updating may decrease the state space dramatically without loss of information. Introduction of decisions on multiple time scales further decreases the size of the state space.

Due to large variation in parameter values among sow herds as a consequence of different production systems and management levels it is concluded that a sow replacement model to be used in a commercial herd must be based on parameter estimates obtained in the same herd.

The multi-level hierarchical approach used in this study is very flexible for adaptations involving decisions and policies at longer as well as shorter time horizons than considered in the present model.

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